

# TEN SIMPLE RULES FOR MATHEMATICAL WRITING

**Dimitri Bertsekas  
M.I.T.**

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# ON WRITING

- **“Easy reading is damn hard writing” (Hawthorne)**
- **“Word-smithing is a much greater percentage of what I am supposed to be doing in life than I would ever have thought” (Knuth)**
- **“I think I can tell someone how to write but I can’t think who would want to listen” (Halmos)**

# WHAT IS MATH WRITING?

- **Writing where mathematics is used as a primary means for expression, deduction, or problem solving.**
- **Examples that are:**
  - **Math papers and textbooks**
  - **Analysis of mathematical models in engineering, physics, economics, finance, etc**
- **Examples that are not:**
  - **Novels, essays, letters, etc**
  - **Experimental/nonmathematical papers and reports**

# WHAT IS DIFFERENT ABOUT MATH WRITING?

- **Math writing blends **two** languages (natural and math)**
  - Natural language is rich and allows for ambiguity
  - Math language is concise and must be unambiguous
- **Math writing requires **slow** reading**
  - Often expresses complex ideas
  - Often must be read and pondered several times
  - Often is used as reference
  - Usually must be read selectively and in pieces

# WHY THIS TALK?

- Experience is something you get only after you need it ...
- One current model: **The conversational style**
  - “Mathematics should be written so that it reads like a conversation between two mathematicians on a walk in the woods” (Halmos)
  - “Talk to your readers as you write” (Strang)
  - Very hard to teach to others (“Effective exposition is not a teachable art. There is no useful recipe ...” Halmos)
  - Controversial (where do proofs start and end? ... I am not sure what the assumptions are ... I can't find what I need ... etc)
- **Instead we will advocate a structured style**
  - Offers specific verifiable rules that students can follow and thesis advisors can check
  - Allows room to develop and improve over time

# SOURCES

- **General style books**
  - Strunk and White, “The Elements of Style” (www)
  - Fowler and Aaron, “The Little Brown Handbook”
  - Venolia, “Write Right!”
- **Halmos, “How to Write Mathematics”**
- **Knuth, et al, “Mathematical Writing” (www)**
- **Kleiman, “Writing a Math Phase Two Paper,” MIT (www)**
- **Krantz, “A Primer of Mathematical Writing”**
- **Higham, “Handbook of Writing for the Mathematical Sciences”**
- **Alley, “The Craft of Scientific Writing”**
- **Thomson, “A Guide for the Young Economist”**

# RULES OF THE GAME

- **Small rules:**
  - Apply to a **single sentence** (e.g., sentence structure rules, mathspeak rules, comma rules, etc)
- **Broad rules:**
  - Apply to the entire document
  - **General** style and writing strategy rules
  - Are **non-verifiable** (e.g., organize, be clear and concise, etc)
- **Composition rules (our focus in this talk):**
  - Relate to how parts of the document connect
  - Apply to **multiple sentences**
  - Are **verifiable**

# SOME EXAMPLES OF SMALL RULES I

- **Break up long blocks of text into simpler ones:**
  - Few lines and verbs per sentence; few sentences per paragraph.
  - **2-3-4 rule:** Consider splitting every sentence of more than 2 lines, every sentence with more than 3 verbs, and every paragraph with more than 4 "long" sentences.
- **Mathspeak should be “readable”**
  - **BAD:** **Let  $k > 0$  be an integer.**
  - **GOOD:** **Let  $k$  be a positive integer** or **Consider an integer  $k > 0$ .**
  - **BAD:** **Let  $x \in \mathbb{R}^n$  be a vector.**
  - **GOOD:** **Let  $x$  be a vector in  $\mathbb{R}^n$**  or **Consider a vector  $x \in \mathbb{R}^n$ .**
- **Don't start a sentence with mathspeak**
  - **BAD:** **Proposition:  $f$  is continuous.**
  - **GOOD:** **Proposition: The function  $f$  is continuous.**



## **SOME EXAMPLES OF SMALL RULES II**

- **Use active voice (“we” is better than “one”)**
- **Minimize “strange” symbols within text**
- **Make proper use of “very,” “trivial,” “easy,” “nice,” “fundamental,” etc**
- **Use abbreviations correctly (e.g., cf., i.e., etc.)**
- **Comma rules**
- **“Which” and “that” rules**
- **... ETC**

# **SOME EXAMPLES OF BROAD RULES**

- **Language rules/goals to strive for: precision, clarity, familiarity, forthrightness, conciseness, fluidity, rhythm**
- **Organizational rules (how to structure your work, how to edit, rewrite, proofread, etc)**
- **“Down with the irrelevant and the trivial” (Halmos)**
- **“Honesty is the best policy” (Halmos)**
- **“Defend your style” (against copyeditors - Halmos)**
- **... ETC**

# THE TEN COMPOSITION RULES

- **Structure rules (break it into digestible pieces)**
  - Organize in segments
  - Write segments linearly
  - Consider a hierarchical development
- **Consistency rules (be boring creatively)**
  - Use consistent notation and nomenclature
  - State results consistently
  - Don't underexplain - don't overexplain
- **Readability rules (make it easy for the reader)**
  - Tell them what you'll tell them
  - Use suggestive references
  - Consider examples and counterexamples
  - Use visualization when possible



# SEGMENTATION PROCESS

- **Examples of segments:**
  - **A mathematical result and its proof**
  - **An example**
  - **Several related results/examples with discussion**
  - **An appendix**
  - **A long abstract**
  - **A conclusions section**
- **A segment should “stand alone” (identifiable start and end, transition material)**
- **Length: 1/2 page to 2-3 pages**

# SEGMENT STRUCTURE

**Title (optional)**



**Transition Material**



**Definitions, Examples  
Arguments, Illustrations**



**Transition Material**

# EXAMPLE OF SEGMENTATION: A SECTION ON PROB. MODELS

- **Sample space - Events** (1 page)
- **Choosing a sample space** (0.5 page)
- **Sequential models** (0.75 page)
- **Probability laws - Axioms** (1.25 page)
- **Discrete models** (2 pages)
- **Continuous models** (1 page)
- **Properties of probability laws** (2 pages)
- **Models and reality** (1.25 page)
- **History of probability** (1 page)

**See Sec. 1.2 of Bertsekas and Tsitsiklis probability book**

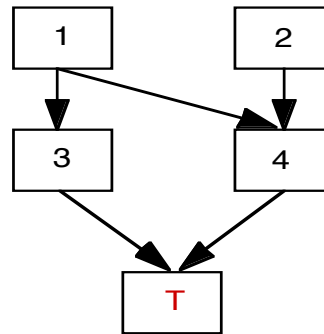
## 2. WRITE SEGMENTS LINEARLY

- Question: What is a good way to order the flow of deduction and dependency?
- General rule: **Arguments should be placed close to where they are used (minimize thinking strain)**
- Similarly, definitions, lemmas, etc, should be placed close to where they are used
- View ordering as an optimization problem
- A **linear/optimal** order is one that positions arguments (definitions, lemmas) so as to minimize the total number of “crossings” over other arguments (definitions, lemmas), subject to the dependency constraints. **Depth-first** order is usually better.



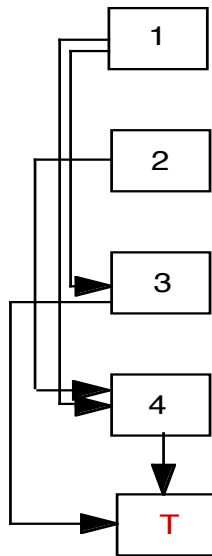
# EXAMPLES OF ORDERING

Dependency  
Graph of  
Arguments



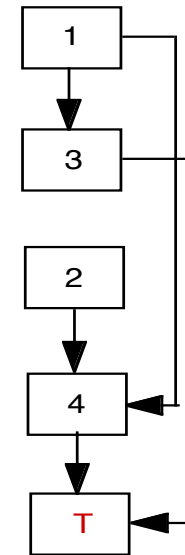
Level 1  
Arguments

Level 2  
Arguments



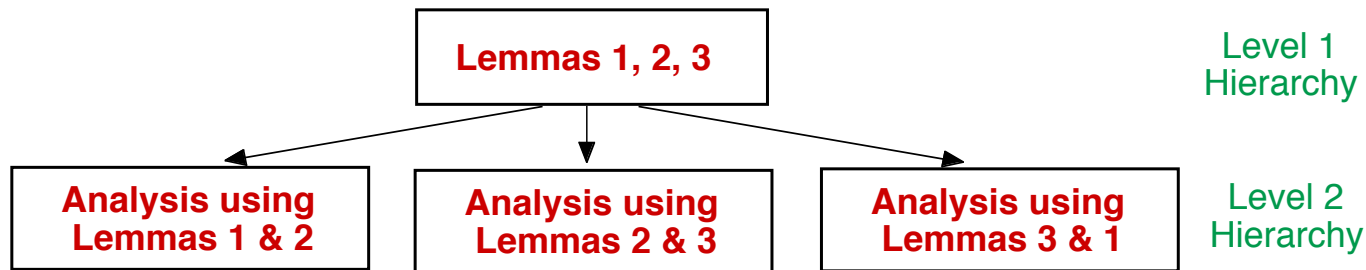
Nonlinear

Linear



## 3. CONSIDER A HIERARCHICAL DEVELOPMENT

- Arguments/results used repeatedly may be placed in special segments for efficiency



- Possibly create special segments for special material (e.g., math background, notation, etc)
- Analogy to subroutines in computer programs

## 4. USE CONSISTENT NOTATION

- Choose a notational style and stick with it
- Examples:
  - Use capitals for random variables, lower case for values
  - Use subscripts for sequences, superscripts for components
- Use suggestive/mnemonic notation. Examples: S for set, f for function, B for ball, etc
- Use simple notation. Example: Try to avoid parenthesized indexes:  $x(m,n)$  vs  $x_{mn}$
- Avoid unnecessary notation:
  - BAD: Let X be a compact subset of a space Y. If f is a continuous real-valued function over X, it attains a minimum over X.
  - GOOD: A continuous real-valued function attains a minimum over a compact set.

## 5. STATE RESULTS CONSISTENTLY

- **Keep your language/format simple and consistent (even boring)**
- **Keep distractions to a minimum; make the interesting content stand out**
- **Use similar format in similar situations**
- **Bad example:**
  - **Proposition 1: If A and B hold, then C and D hold.**
  - **Proposition 2: C' and D' hold, assuming that A' and B' are true.**
- **Good example:**
  - **Proposition 1: If A and B hold, then C and D hold.**
  - **Proposition 2: If A' and B' hold, then C' and D' hold.**

## 6. DON'T OVEREXPLAIN - DON'T UNDEREXPLAIN

- Choose a **target audience level** of expertise/ background (e.g., undergraduate, 1st year graduate, research specialist, etc)
- Aim your math to that level; don't go much over or under
- Explain potentially unfamiliar material in separate segment(s)
- Consider the use of appendixes for background or difficult/specialized material

## 7. TELL THEM WHAT YOU' LL TELL THEM

- **Keep the reader informed** about where you are and where you are going
- Start each segment with a short introduction and perhaps a road map
- Don't string together seemingly aimless statements and surprise the reader with “we have thus proved so and so”
- Announce your intentions/results, e.g., “It turns out that so-and-so is true. To see this, note ...”
- Tell them what you told them

## 8. USE SUGGESTIVE REFERENCES

- Frequent numbered equation/proposition referencing is a **cardinal sin**
- It causes page flipping, wastes the reader's time, and breaks concentration
- Refer to equations/results/assumptions by content/name (in addition to number), e.g., Bellman's equation, weak duality theorem, etc
- Repeat simple math expressions
- Remind the reader of unusual notation, and earlier analysis
- Dare to be repetitive (but don't overdo it)

## **9. CONSIDER EXAMPLES AND COUNTEREXAMPLES**

- **“Even a simple example will get three-quarters of an idea across” (Ullman)**
- **Examples should have some spark, i.e., aim at something the reader may have missed**
- **Illustrate definitions/results with examples that clarify the boundaries of applicability**
- **Use counterexamples to clarify the limitations of the analysis, and the need for the assumptions**



## **10. USE VISUALIZATION WHEN POSSIBLE**

- **“A picture is worth a thousand words”**
- **Keep figures simple and uncluttered**
- **Use substantial captions**
- **Captions should reinforce and augment the text, not repeat it**
- **Use a figure to illustrate the main idea of a proof/argument with no constraint of math formality**
- **Prefer graphs over tables**

**THE END**

**“Bad thinking never produces good writing”  
(Lamport)**

**Good writing promotes good thinking ...**