MTH 100 — College Algebra
Essex County College — Division of Mathematics
Sample Review Questions[1] — Created June 6, 2011

Math 100, Introductory College Mathematics, covers the mathematical content listed below. In order to *place out* of Math 100 or to prepare for the final examination in this course, one should be extremely comfortable with all of these items.

NOTE: Calculators are permitted in this course, but should only be used when appropriate. The content listed below is covered in chapters 1 through 8 and 10 in *Algebra for College Students*, Custom Version for Essex County College, Second Edition, by Bittinger & Ellenbogen (published by Addison Wesley, Inc.), the current textbook used in Math 100.

- Simplifying expressions involving negative integer and rational exponents.
- Simplifying, adding, subtracting, multiplying, and dividing rational expressions, radical expressions, and complex numbers.
- Multiplying and dividing polynomials.
- Factoring polynomials including greatest common factors (GCF), differences of squares, differences and sums of cubes, and quadratic trinomials.
- Evaluating expressions involving function notation.
- Finding the domains of functions.
- Solving equations, including ones containing rational or radical terms.
- Solving literal equations or formulas for a specified variable in terms of other variables.
- Solving inequalities and systems of equations.
- Solving quadratic equations by using the zero-product rule, the quadratic formula, and the square-root method in conjunction with completing the square.
- Solving application problems (including, but not limited to, total value, mixture, motion, work, area, and determine-the-number problems).
- Finding equations of lines through given points, with given slopes, perpendicular or parallel to given lines, and/or with given intercepts.
- Finding slopes and x and y-intercepts of lines.
- Finding vertices and lines of symmetry of parabolas and centers and radii of circles.
- Finding equations of circles with given centers and radii.
- Graphing lines, parabolas, and circles.
- Finding the distance between two points.

[1] This document was prepared by Ron Bannon using LaTeX 2_ε and is a slight modification of Susan Gaulden's *MTH 100 Content* worksheet.

Review Problem Set

At Essex County College you should be prepared to show all work *clearly* and in *order*, ending your work by $\boxed{\text{boxing the answer.}}$ Furthermore, justify your answers algebraically whenever possible. These questions are for review only, and placement tests are not limited to these problems alone. Solutions and work are provided for each question. Please feel free to email rbannon@mac.com with questions or comments pertaining to this document.[2]

1. Simplify.

   (a) $\left(3x^{-2}y^3z^0\right)^{-2}$

   Solution:

$$\left(3x^{-2}y^3z^0\right)^{-2} = \left(\frac{3y^3}{x^2}\right)^{-2} = \left(\frac{x^2}{3y^3}\right)^2 = \boxed{\frac{x^4}{9y^6}}$$

   (b) $\dfrac{24a^3b^5c^{-1}}{16a^{-2}c}$

   Solution:

$$\frac{24a^3b^5c^{-1}}{16a^{-2}c} = \boxed{\frac{3a^5b^5}{2c^2}}$$

   (c) $\left(\dfrac{3m^{-4}n}{6mn^2}\right)^{-3}$

   Solution:

$$\left(\frac{3m^{-4}n}{6mn^2}\right)^{-3} = \left(\frac{1}{2m^5n}\right)^{-3} = \left(2m^5n\right)^3 = \boxed{8m^{15}n^3}$$

2. Simplify.

   (a) $\left(25x^2y^4\right)^{\frac{1}{2}}$

   Solution:

$$\left(25x^2y^4\right)^{\frac{1}{2}} = \boxed{5\,|x|\,y^2}$$

   (b) $\left(8a^6b^9c^3\right)^{\frac{2}{3}}$

   Solution:

$$\left(8a^6b^9c^3\right)^{\frac{2}{3}} = \boxed{4a^4b^6c^2}$$

---

[2]These are sample problems and you should not limit your study to just these problems.

3. Simplify

(a) $$\frac{x^2-5x+6}{x^2+2x-8}$$

Solution:

$$\frac{x^2-5x+6}{x^2+2x-8} = \frac{(x-2)(x-3)}{(x+4)(x-2)} = \boxed{\frac{x-3}{x+4}, \quad x \neq 2}$$

(b) $$\frac{6x-7}{4x-5} + \frac{2x-3}{4x-5}$$

Solution:

$$\frac{6x-7}{4x-5} + \frac{2x-3}{4x-5} = \frac{6x-7+2x-3}{4x-5} = \frac{8x-10}{4x-5} = \frac{2(4x-5)}{(4x-5)} = \boxed{2, \quad x \neq \frac{5}{4}}$$

(c) $$\frac{3}{x+1} + \frac{5x}{x+2} - \frac{1}{x^2+3x+2}$$

Solution:

$$\begin{aligned}
\frac{3}{x+1} + \frac{5x}{x+2} - \frac{1}{x^2+3x+2} &= \frac{3}{(x+1)} + \frac{5x}{(x+2)} - \frac{1}{(x+1)(x+2)} \\
&= \frac{3(x+2)}{(x+1)(x+2)} + \frac{5x(x+1)}{(x+1)(x+2)} - \frac{1}{(x+1)(x+2)} \\
&= \frac{3x+6+5x^2+5x-1}{(x+1)(x+2)} \\
&= \boxed{\frac{5x^2+8x+5}{(x+1)(x+2)}}
\end{aligned}$$

(d) $$\frac{x^2-4}{3x+1} \cdot \frac{6x+2}{x^2-7x+10}$$

Solution:

$$\begin{aligned}
\frac{x^2-4}{3x+1} \cdot \frac{6x+2}{x^2-7x+10} &= \frac{(x+2)(x-2)}{(3x+1)} \cdot \frac{2(3x+1)}{(x-2)(x-5)} \\
&= \frac{(x+2)}{1} \cdot \frac{2}{(x-5)}, \quad x \neq 2, \quad x \neq -\frac{1}{3} \\
&= \boxed{\frac{2x+4}{x-5}, \quad x \neq 2, \quad x \neq -\frac{1}{3}}
\end{aligned}$$

(e) $\dfrac{x^2+4x+3}{x^2-3x-10} \div \dfrac{x^2-x-12}{x^2-9x+20}$

Solution:

$$\begin{aligned}
\frac{x^2+4x+3}{x^2-3x-10} \div \frac{x^2-x-12}{x^2-9x+20} &= \frac{x^2+4x+3}{x^2-3x-10} \cdot \frac{x^2-9x+20}{x^2-x-12} \\
&= \frac{(x+3)(x+1)}{(x+2)(x-5)} \cdot \frac{(x-5)(x-4)}{(x+3)(x-4)} \\
&= \boxed{\frac{x+1}{x+2}, \quad x \neq -3, \quad x \neq 4, \quad x \neq 5}
\end{aligned}$$

4. Simplify.

(a) $\dfrac{\dfrac{2}{9x}+\dfrac{1}{3}}{x-\dfrac{4}{9x}}$

Solution:

$$\begin{aligned}
\frac{\frac{2}{9x}+\frac{1}{3}}{x-\frac{4}{9x}} &= \frac{\frac{2}{9x}+\frac{1}{3}}{x-\frac{4}{9x}} \cdot \frac{9x}{9x} \\
&= \frac{2+3x}{9x^2-4}, \quad x \neq 0 \\
&= \frac{(2+3x)}{(3x+2)(3x-2)}, \quad x \neq 0 \\
&= \boxed{\frac{1}{3x-2}, \quad x \neq 0, \quad x \neq -\frac{2}{3}}
\end{aligned}$$

(b) $\dfrac{\dfrac{2}{x-3}-\dfrac{3}{2x+1}}{\dfrac{1}{2x+1}+\dfrac{5}{x-3}}$

Solution:

$$\begin{aligned}
\frac{\frac{2}{x-3}-\frac{3}{2x+1}}{\frac{1}{2x+1}+\frac{5}{x-3}} &= \frac{\frac{2}{x-3}-\frac{3}{2x+1}}{\frac{1}{2x+1}+\frac{5}{x-3}} \cdot \frac{(x-3)(2x+1)}{(x-3)(2x+1)} \\
&= \frac{2(2x+1)-3(x-3)}{1(x-3)+5(2x+1)}, \quad x \neq 3, \quad x \neq -\frac{1}{2} \\
&= \boxed{\frac{x+11}{11x+2}, \quad x \neq 3, \quad x \neq -\frac{1}{2}}
\end{aligned}$$

5. Simplify.

(a) $\sqrt[4]{32m^7n^8p^{12}}$

Solution:

$$\sqrt[4]{32m^7n^8p^{12}} = \boxed{2n^2\left|mp^3\right|\sqrt[4]{2m^3}}$$

(b) $\sqrt{\dfrac{80x^3y}{5xy^5}}$

Solution: Note, the restriction here is that $x \neq 0$ and $y \neq 0$.

$$\sqrt{\frac{80x^3y}{5xy^5}} = \sqrt{\frac{16x^2}{y^4}} = \boxed{\frac{4\left|x\right|}{y^2}}$$

(c) $3\sqrt[4]{7} - 5\sqrt{2} + \sqrt[4]{7} + 2\sqrt{2}$

Solution:

$$3\sqrt[4]{7} - 5\sqrt{2} + \sqrt[4]{7} + 2\sqrt{2} = \boxed{4\sqrt[4]{7} - 3\sqrt{2}}$$

(d) $\sqrt{48} + 3\sqrt{27} - 4\sqrt{12}$

Solution:

$$\sqrt{48} + 3\sqrt{27} - 4\sqrt{12} = 4\sqrt{3} + 9\sqrt{3} - 8\sqrt{3} = \boxed{5\sqrt{3}}$$

(e) $\sqrt{40}\sqrt{30}$

Solution:

$$\sqrt{40}\sqrt{30} = \sqrt{1200} = \sqrt{3 \cdot 4 \cdot 100} = \boxed{20\sqrt{3}}$$

(f) $\sqrt[5]{4a^4bc^4} \cdot \sqrt[5]{8a^3b^2c}$

Solution:

$$\sqrt[5]{4a^4bc^4} \cdot \sqrt[5]{8a^3b^2c} = \sqrt[5]{32a^7b^3c^5} = \boxed{2ac\sqrt[5]{a^2b^3}}$$

(g) $\dfrac{\sqrt{20xy^5}}{\sqrt{5x^3y}}$

Solution: The restriction here is that $x > 0$ and $y > 0$.

$$\frac{\sqrt{20xy^5}}{\sqrt{5x^3y}} = \sqrt{\frac{20xy^5}{5x^3y}} = \sqrt{\frac{4y^4}{x^2}} = \boxed{\frac{2y^2}{x}}$$

(h) $\dfrac{3-\sqrt{2}}{5+\sqrt{2}}$

Solution:

$$\frac{3-\sqrt{2}}{5+\sqrt{2}} = \frac{3-\sqrt{2}}{5+\sqrt{2}} \cdot \frac{5-\sqrt{2}}{5-\sqrt{2}} = \frac{17-8\sqrt{2}}{25-2} = \boxed{\frac{17-8\sqrt{2}}{23}} = \boxed{\frac{17}{23} - \frac{8\sqrt{2}}{23}}$$

6. Rationalize the denominator $\sqrt{\dfrac{9}{20x^2y}}$.

Solution: The restriction here is that $x \neq 0$ and $y > 0$.

$$\sqrt{\frac{9}{20x^2y}} = \frac{\sqrt{9}}{\sqrt{20x^2y}} = \frac{3}{2\,|x|\,\sqrt{5y}} = \frac{3}{2\,|x|\,\sqrt{5y}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}} = \boxed{\frac{3\sqrt{5y}}{10\,|x|\,y}}$$

7. Simplify.

(a) $3\sqrt{-4} + 5\sqrt{-16}$

Solution:

$$3\sqrt{-4} + 5\sqrt{-16} = 3 \cdot 2i + 5 \cdot 4i = 6i + 20i = \boxed{26i}$$

(b) $(3+2i) + (5-6i) - (1+4i)$

Solution:

$$(3+2i) + (5-6i) - (1+4i) = \boxed{7-8i}$$

(c) $5i\,(7-4i)$

Solution:

$$5i\,(7-4i) = 35i - 20i^2 = 35i + 20 = \boxed{20+35i}$$

(d) $(4+i)\,(3-2i)$

Solution:

$$(4+i)\,(3-2i) = 12 - 8i + 3i - 2i^2 = 12 - 5i + 2 = \boxed{14-5i}$$

(e) $\dfrac{4+i}{3-2i}$

Solution:

$$\frac{4+i}{3-2i} = \frac{4+i}{3-2i} \cdot \frac{3+2i}{3+2i} = \frac{10+11i}{9-4i^2} = \frac{10+11i}{13} = \boxed{\frac{10}{13} + \frac{11}{13}i}$$

8. Simplify.

   (a) $\left(x^2-2x+4\right)\left(3x^2+5x-2\right)$

   Solution:

$$\begin{aligned}\left(x^2-2x+4\right)\left(3x^2+5x-2\right) &= 3x^4+5x^3-2x^2-6x^3-10x^2+4x+12x^2+20x-8\\ &= \boxed{3x^4-x^3+24x-8}\end{aligned}$$

   (b) $\dfrac{2x^2-5x-12}{x-4}$

   Solution:

$$\frac{2x^2-5x-12}{x-4}=\frac{(2x+3)(x-4)}{(x-4)}=\boxed{2x+3, \quad x\neq 4}$$

   (c) $\left(3x^2+7x-4\right)\div(x+2)$

   Solution: Found by long division.

$$\boxed{3x+1-\frac{6}{x+2}}$$

9. Factor.

   (a) $5x^3y^2-15x^2y^4+20x^3y^3$

   Solution:

$$5x^3y^2-15x^2y^4+20x^3y^3=\boxed{5x^2y^2\left(x-3y^2+4xy\right)}$$

   (b) m^4-16

   Solution:

$$m^4-16=\left(m^2-4\right)\left(m^2+4\right)=\boxed{(m-2)(m+2)\left(m^2+4\right)}$$

   (c) $27x^3-1$

   Solution:

$$27x^3-1=\boxed{(3x-1)\left(9x^2+3x+1\right)}$$

   (d) $15x^2+11x+2$

   Solution:

$$15x^2+11x+2=\boxed{(5x+2)(3x+1)}$$

(e) $x^3 + 64$

Solution:

$$\boxed{x^3 + 64 = (x+4)\left(x^2 - 4x + 16\right)}$$

10. Evaluate.

(a) $f(-3)$ given that $f(x) = \dfrac{3x-2}{x^2+5}$.

Solution:

$$f(-3) = \frac{3(-3)-2}{(-3)^2+5} = \frac{-9-2}{9+5} = \boxed{-\frac{11}{14}}$$

(b) $g(2)$ given that $g(x) = \sqrt{x^3 - x + 10}$.

Solution:

$$g(2) = \sqrt{2^3 - 2 + 10} = \sqrt{8-2+10} = \sqrt{16} = \boxed{4}$$

(c) $h(0)$ given that $h(x) = 4x + \sqrt[3]{x^2+8}$.

Solution:

$$h(0) = 4 \cdot 0 + \sqrt[3]{0^2+8} = \sqrt[3]{8} = \boxed{2}$$

11. Find the domain of each function.

(a) $f(x) = \dfrac{5}{x}$

Solution: $\boxed{\mathbb{R}, x \neq 0}$

(b) $g(x) = \dfrac{3x+5}{2x-1}$

Solution: $\boxed{\mathbb{R}, x \neq \frac{1}{2}}$

(c) $h(x) = 3x^2 - 2x + 7$

Solution: $\boxed{\mathbb{R}}$

12. Solve.

(a) $2x - 8 = 5(x + 4) - x$

Solution:

$$\begin{aligned} 2x - 8 &= 5(x + 4) - x \\ 2x - 8 &= 5x + 20 - x \\ 2x - 8 &= 4x + 20 \\ -8 - 20 &= 4x - 2x \\ -28 &= 2x \\ -14 &= x \end{aligned}$$

So, $\boxed{x = -14}$.

(b) $\dfrac{6}{x} + \dfrac{4}{x} = 5$

Solution:

$$\begin{aligned} \frac{6}{x} + \frac{4}{x} &= 5 \\ x \cdot \frac{6}{x} + x \cdot \frac{4}{x} &= x \cdot 5 \\ 6 + 4 &= 5x \\ 10 &= 5x \\ 2 &= x \end{aligned}$$

So, $\boxed{x = 2}$.

(c) $\dfrac{3}{x + 4} = \dfrac{2}{x - 3}$

Solution:

$$\begin{aligned} \frac{3}{x + 4} &= \frac{2}{x - 3} \\ (x + 4)(x - 3) \cdot \frac{3}{(x + 4)} &= (x + 4)(x - 3) \cdot \frac{2}{(x - 3)} \\ (x - 3) \cdot 3 &= (x + 4) 2 \\ 3x - 9 &= 2x + 8 \\ 3x - 2x &= 9 + 8 \\ x &= 17 \end{aligned}$$

So, $\boxed{x = 17}$.

(d) $\dfrac{50}{x-2} - \dfrac{16}{x} = \dfrac{30}{x}$

Solution:

$$\begin{aligned}
\frac{50}{x-2} - \frac{16}{x} &= \frac{30}{x} \\
x(x-2) \cdot \frac{50}{x-2} - x(x-2) \cdot \frac{16}{x} &= x(x-2) \cdot \frac{30}{x} \\
x \cdot 50 - (x-2) \cdot 16 &= (x-2)\,30 \\
50x - 16x + 32 &= 30x - 60 \\
34x + 32 &= 30x - 60 \\
34x - 30x &= -32 - 60 \\
4x &= -92 \\
x &= -23
\end{aligned}$$

So, $\boxed{x = -23}$.

(e) $\sqrt{x+7} + 5 = x$

Solution:

$$\begin{aligned}
\sqrt{x+7} + 5 &= x \\
\sqrt{x+7} &= x - 5 \\
\left(\sqrt{x+7}\right)^2 &= (x-5)^2 \\
x + 7 &= x^2 - 10x + 25 \\
0 &= x^2 - 11x + 18 \\
0 &= (x-2)(x-9)
\end{aligned}$$

It appears that $x = 2$ and $x = 9$ are solutions, however, $x = 2$ is extraneous (*i.e.* doesn't work), so $\boxed{x = 9}$.

(f) $\sqrt[3]{x-2} = -3$

Solution:

$$\begin{aligned}
\sqrt[3]{x-2} &= -3 \\
\left(\sqrt[3]{x-2}\right)^3 &= (-3)^3 \\
x - 2 &= -27 \\
x &= -25
\end{aligned}$$

So, $\boxed{x = -25}$.

13. Solve.

(a) $A = P(1 + rt)$ for r.

Solution:

$$\begin{aligned} A &= P(1+rt) \\ A &= P + Prt \\ A - P &= Prt \\ \frac{A-P}{Pt} &= r \end{aligned}$$

So, $\boxed{r = \dfrac{A-P}{Pt}}$ or $\boxed{r = \dfrac{A}{Pt} - \dfrac{1}{t}}$.

(b) $v = \dfrac{d_1 + d_2}{t}$ for d_2

Solution:

$$\begin{aligned} v &= \frac{d_1 + d_2}{t} \\ vt &= d_1 + d_2 \\ vt - d_1 &= d_2 \end{aligned}$$

So, $\boxed{d_2 = vt - d_1}$.

(c) $K = \dfrac{rt}{r-t}$ for t

Solution:

$$\begin{aligned} K &= \frac{rt}{r-t} \\ K\left(r-t\right) &= rt \\ Kr - Kt &= rt \\ Kr &= rt + Kt \\ Kr &= t\left(r+K\right) \\ \frac{Kr}{r+K} &= t \end{aligned}$$

So, $\boxed{y = \dfrac{Kr}{r+K}}$.

14. Solve.

(a) $4x - 7 > x + 11$

Solution:

$$\begin{aligned} 4x - 7 &> x + 11 \\ 4x - x &> 7 + 11 \\ 3x &> 18 \\ x &> 6 \end{aligned}$$

So, $\boxed{x > 6}$, and in interval notation, $\boxed{(6,\ \infty)}$.

(b) $3x - 2(x - 4) \leq 2x + 5(x + 1)$

Solution:

$$\begin{aligned} 3x - 2(x - 4) &\leq 2x + 5(x + 1) \\ 3x - 2x + 8 &\leq 2x + 5x + 5 \\ x + 8 &\leq 7x + 5 \\ 8 - 5 &\leq 7x - x \\ 3 &\leq 6x \\ \frac{1}{2} &\leq x \end{aligned}$$

So, $\boxed{x \geq \frac{1}{2}}$, and in interval notation, $\boxed{\left[\frac{1}{2},\ \infty\right)}$.

15. Solve.

(a) $\begin{cases} x + 2y = -1 \\ y = 2x + 7 \end{cases}$

Solution: Using the method of substitution.

$$\begin{aligned} x + 2y &= -1 \\ x + 2(2x + 7) &= -1 \\ x + 4x + 14 &= -1 \\ 5x + 14 &= -1 \\ 5x &= -15 \\ x &= -3 \end{aligned}$$

So, $\boxed{x = -3}$ and since $y = 2x + 7 = 2(-3) + 7 = 1$, $\boxed{y = 1}$.

(b) $\begin{cases} 3x - 2y = 7 \\ 2x + 5y = -8 \end{cases}$

Solution: Using the method of elimination, by multiplying the first row by -2 and the second row by 3, and then adding the two equations together.

$$\begin{cases} -2(3x - 2y) = -2(7) \\ 3(2x + 5y) = 3(-8) \end{cases} \Rightarrow \begin{cases} -6x + 4y = -14 \\ 6x + 15y = -24 \end{cases} \Rightarrow 19y = -38 \Rightarrow y = -2.$$

Using $y = -2$ to back-substitute into any of these equations to find x.

$$3x - 2y = 7 \Rightarrow 3x + 4 = 7 \Rightarrow 3x = 3 \Rightarrow x = 1$$

So, finally, the solution, in ordered-pair notation, is $\boxed{(1,\ -2)}$

16. Solve.

(a) $x^2 - 9 = 0$

Solution: Using the square-root rule.

$$x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm\sqrt{9} \Rightarrow \boxed{x = \pm 3}$$

(b) $x^2 = -2x$

Solution:

$$\begin{aligned} x^2 &= -2x \\ x^2 + 2x &= 0 \\ x(x+2) &= 0 \end{aligned}$$

So, by the zero product rule, $\boxed{x = 0 \text{ or } x = -2}$.

(c) $3x^2 - x - 10 = 0$

Solution:

$$\begin{aligned} 3x^2 - x - 10 &= 0 \\ (3x+5)(x-2) &= 0 \end{aligned}$$

So, by the zero product rule, $\boxed{x = -\frac{5}{3} \text{ or } x = 2}$.

(d) $x^2 - 8x + 9 = 0$

Solution: Since $x^2 - 8x + 9$ is not factorable over the rational numbers, you'll need to either complete the square or use the quadratic formula. Here I am using the technique of completing the square.

$$\begin{aligned} x^2 - 8x + 9 &= 0 \\ x^2 - 8x &= -9 \\ x^2 - 8x + 16 &= -9 + 16 \\ (x-4)^2 &= 7 \\ x - 4 &= \pm\sqrt{7} \\ x &= 4 \pm \sqrt{7} \end{aligned}$$

So, $\boxed{x = 4 + \sqrt{7} \text{ or } x = 4 - \sqrt{7}}$.

(e) $2x^2 - 6x = 5$

Solution: Since $2x^2 - 6x - 5$ is not factorable over the rational numbers, you'll need to either complete the square or use the quadratic formula. Here I am using the quadratic formula.

$$2x^2 - 6x = 5 \quad \Rightarrow \quad 2x^2 - 6x - 5 = 0 \quad \Rightarrow \quad x = \frac{6 \pm \sqrt{36 - 4(2)(-5)}}{4} = \frac{6 \pm \sqrt{76}}{4} = \frac{6 \pm 2\sqrt{19}}{4}$$

So, $\boxed{x = \frac{3 + \sqrt{19}}{2} \text{ or } x = \frac{3 - \sqrt{19}}{2}}$.

17. Find the equation of the line through the point $(0, 5)$ with slope $-2/3$. Graph the line.

Solution: Using the point slope form.

$$\boxed{y - 5 = -\frac{2}{3}(x - 0)}$$

Or, the slope intercept form.

$$\boxed{y = -\frac{2}{3}x + 5}$$

Or, standard form.

$$\boxed{2x + 3y = 15}$$

Here's the graph.

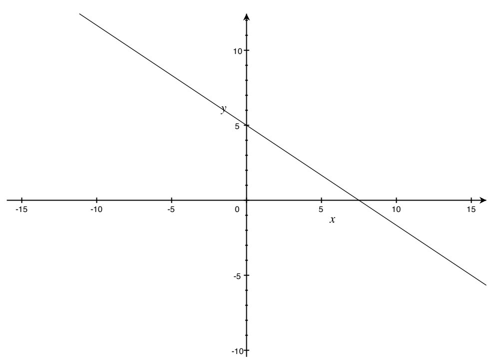

Figure 1: Graph of $2x + 3y = 15$.

18. Find the equation of the the line through the point $(-2,\ 5)$ line with slope -1. Graph the line.

Solution: Using the point slope form.

$$\boxed{y - 5 = -1\,(x + 2)}$$

Or, the slope intercept form.

$$\boxed{y = -x + 3}$$

Or, standard form.

$$\boxed{x + y = 3}$$

Here's the graph.

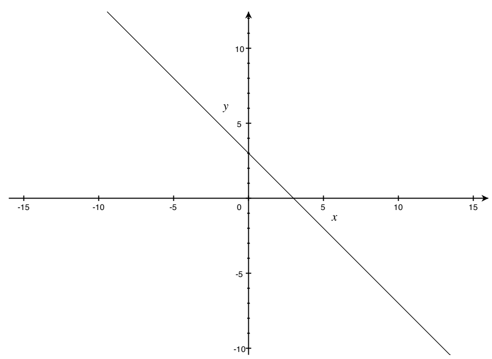

Figure 2: Graph of $x + y = 3$.

19. Find the equation of the line through the points $(4,\ 1)$ and $(2,\ -1)$. Graph the line.

Solution: First you'll need to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{2 - 4} = \frac{-2}{-2} = 1$$

Using the point slope form with the point $(4,\ 1)$.

$$\boxed{y - 1 = 1 \cdot (x - 4)}$$

Using the point slope form with the point $(2,\ -1)$.

$$\boxed{y + 1 = 1 \cdot (x - 2)}$$

Or, the slope intercept form.

$$\boxed{y = x - 3}$$

Or, standard form.

$$\boxed{x - y = 3}$$

Here's the graph.

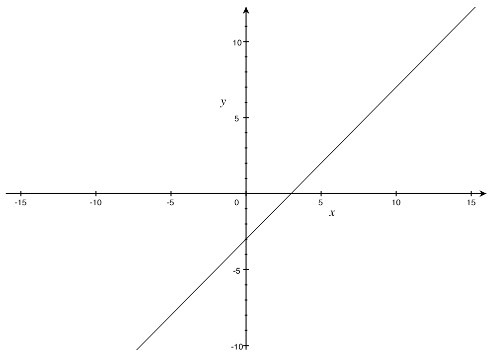

Figure 3: Graph of $x - y = 3$.

20. Find the equation of the line through the point (0, 3) that is perpendicular to the line $2x - y = 3$. Graph the line.

Solution: The slope of the line whose equation is $2x - y = 3$ can be found easily by solving for y.

$$2x - y = 3 \quad \Rightarrow \quad y = 2x - 3$$

So the slope of the line perpendicular to this line is $-1/2$. Using the point slope form.

$$\boxed{y - 3 = -\frac{1}{2}(x - 0)}$$

Or, the slope intercept form.

$$\boxed{y = -\frac{1}{2}x + 3}$$

Or, standard form.

$$\boxed{x + 2y = 6}$$

Here's the graph.

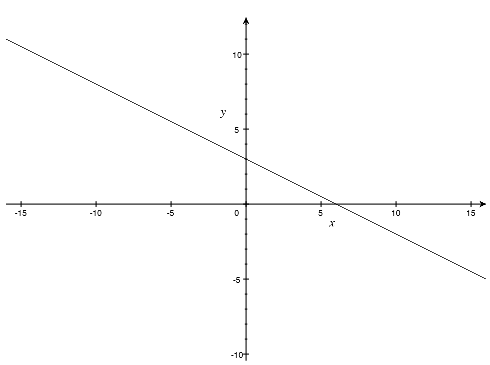

Figure 4: Graph of $x + 2y = 6$.

21. Find the equation of the line through the point $(1,\ 4)$ that is parallel to the line $x - 2y = 5$. Graph the line.

Solution: The slope of the line whose equation is $x - 2y = 5$ can be found easily by solving for y.

$$x - 2y = 5 \quad \Rightarrow \quad y = \frac{1}{2}x - \frac{5}{2}$$

So the slope of the line parallel to this line is 1/2. Using the point slope form.

$$\boxed{y - 4 = \frac{1}{2}(x - 1)}$$

Or, the slope intercept form.

$$\boxed{y = \frac{1}{2}x + \frac{7}{2}}$$

Or, standard form.

$$\boxed{-x + 2y = 7}$$

Here's the graph.

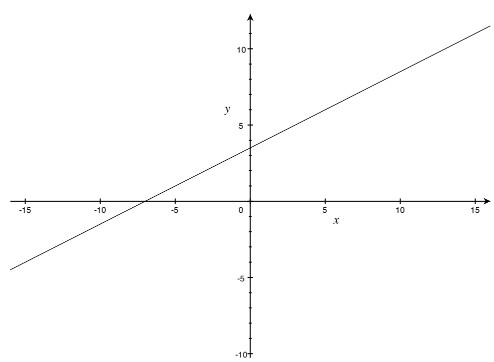

Figure 5: Graph of $-x + 2y = 7$.

22. Find the equation of the line through the point $(0,\ 4)$ and $(-3,\ 0)$. Graph the line.

Solution: First you'll need to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{-3 - 0} = \frac{-4}{-3} = \frac{4}{3}$$

Using the point slope form with the point $(0,\ 4)$.

$$\boxed{y - 4 = \frac{4}{3} \cdot (x - 0)}$$

Using the point slope form with the point $(-3,\ 0)$.

$$\boxed{y - 0 = \frac{4}{3} \cdot (x + 3)}$$

Or, the slope intercept form.

$$\boxed{y = \frac{4}{3}x + 4}$$

Or, standard form.

$$\boxed{-4x + 3y = 12}$$

Here's the graph.

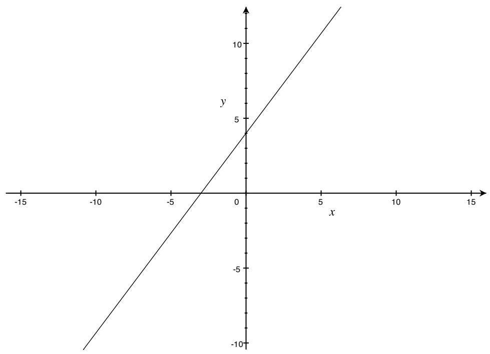

Figure 6: Graph of $-4x + 3y = 12$.

23. Find the slope of the line given by $3x - 4y = 6$.

    Solution: The slope of the line whose equation is $3x - 4y = 6$ can be found easily by solving for y.

    $$3x - 4y = 6 \quad \Rightarrow \quad y = \frac{3}{4}x - \frac{3}{2}$$

    So the slope of this line is $\boxed{\frac{3}{4}}$.

24. Find the slope of the line through the points $(4,\ 1)$ and $(-2,\ 5)$.

    Solution: Using the formula.

    $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{-2-4} = \frac{4}{-6} = \boxed{-\frac{2}{3}}$$

25. Find the x-intercept and the y-intercept of the line given by $3x - 4y = 12$.

    Solution: To find the x-intercept, set $y = 0$ and solve for x.

    $$3x = 12 \quad \Rightarrow \quad x = 4$$

    So the x-intercept is $\boxed{(4,\ 0)}$. To find the y-intercept, set $x = 0$ and solve for y.

    $$-4y = 12 \quad \Rightarrow \quad y = -3$$

    So the y-intercept is $\boxed{(0,\ -3)}$.

26. Determine the vertex and the line of symmetry of the parabola given by $y = 2\left(x-1\right)^2 - 4$. Graph the parabola.

Solution: By inspection the vertex is $\boxed{(1,\ -4)}$ and the axis-of-symmetry is $\boxed{x = 1}$.

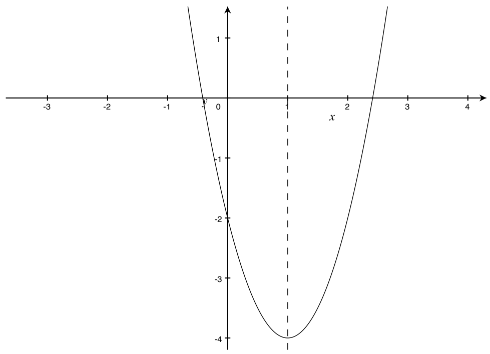

Figure 7: Graph of $y = 2\left(x-1\right)^2 - 4$.

27. Determine the vertex and the line of symmetry of the parabola given by $y = x^2 - 4x + 3$. Graph the parabola.

Solution: Rewrite into standard form.

$$\begin{aligned} y &= x^2 - 4x + 3 \\ y &= \left(x^2 - 4x + 4\right) - 4 + 3 \\ y &= \left(x^2 - 2\right)^2 - 1 \end{aligned}$$

By inspection the vertex is $\boxed{(2,\ -1)}$ and the axis-of-symmetry is $\boxed{x = 2}$.

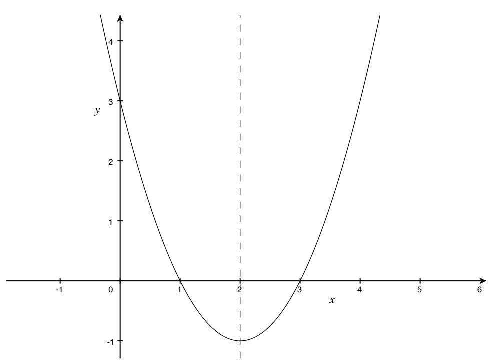

Figure 8: Graph of $y = x^2 - 4x + 3$.

28. Determine the equation of the circle with center $(1,\ -2)$ and radius 3. Graph the circle.

Solution:

$$\boxed{(x-1)^2+(y+2)^2=9}$$

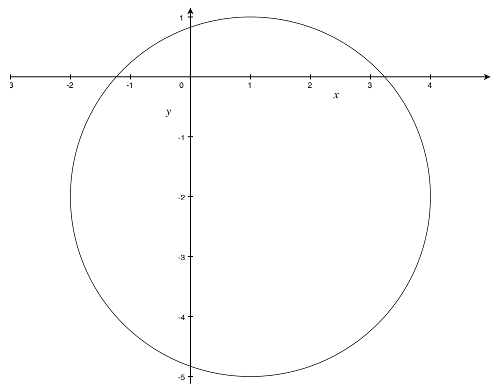

Figure 9: Graph of $(x-1)^2+(y+2)^2=9$.

29. Determine the center and radius of the circle given by $(x+1)^2+(y-3)^2=4$. Graph the circle.

Solution: The center is $\boxed{(-1,\ 3)}$ and radius $\boxed{2}$

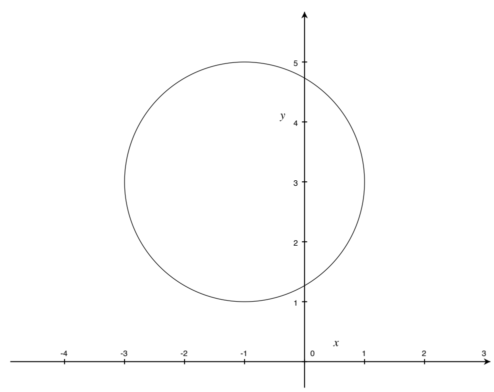

Figure 10: Graph of $(x+1)^2+(y-3)^2=4$.

30. Determine the center and radius of the circle given by $x^2 + y^2 + 6x + 4y + 12 = 0$. Graph the circle.

Solution: Rewrite into standard form.

$$\begin{aligned} x^2 + y^2 + 6x + 4y + 12 &= 0 \\ x^2 + 6x + y^2 + 4y &= -12 \\ x^2 + 6x + 9 + y^2 + 4y + 4 &= -12 + 9 + 4 \\ (x+3)^2 + (x+2)^2 &= 1 \end{aligned}$$

The center is $\boxed{(-3,\ -2)}$ and radius $\boxed{1}$

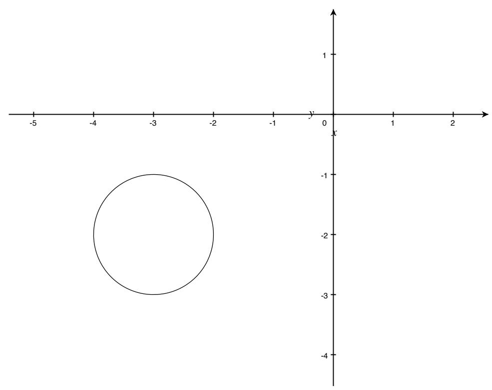

Figure 11: Graph of $x^2 + y^2 + 6x + 4y + 12 = 0$.

31. Determine the distance between the points $(1,\ -5)$ and $(5,\ -2)$.

Solution: Using the distance formula.

$$\sqrt{(5-1)^2 + (-2+5)^2} = \sqrt{16+9} = \sqrt{25} = \boxed{5}$$

32. Determine the distance between the points $(0,\ 6)$ and $(4,\ -1)$.

Solution:Using the distance formula.

$$\sqrt{(4-0)^2 + (-1-6)^2} = \sqrt{16+49} = \boxed{\sqrt{65}}$$

33. The sum of two numbers is 18. The difference between four times the smaller number and seven is equal to the sum of two times the larger number and five. Find the numbers.

    Solution: Let s be the smaller number and l be the larger number. Resulting in this system of equations:

$$\begin{aligned} s + l &= 18 \\ 4s - 7 &= 2l + 5. \end{aligned}$$

    Solving this system, $s = 8$ and $l = 10$. So $\boxed{\text{the two numbers are 8 and 10}}$.

34. The square of a negative number is fifteen more than twice the negative number. Find the number.

    Solution: Let x be the negative number. Resulting equation is:

$$x^2 = 2x + 15.$$

    Solving this equation gives two solutions, $x = 5$ or $x = -3$. So $\boxed{\text{the number is } -3}$.

35. The degree measures of the angles in a triangle are three consecutive integers. Find the measure of the angles.

    Solution: Let x, $x+1$ and $x+2$ be three consecutive integers. Resulting in this equation:

$$x + x + 1 + x + 2 = 180°.$$

    Solving this equation, $x = 59°$. So $\boxed{\text{the measures of the angles are } 59°\text{, } 60°\text{, and } 61°}$.

36. The perimeter of a rectangle is 50 meters. The width of the rectangle is five meters less than the length. Find the length and width of the rectangle.

    Solution: Let l be the measure of the length of the rectangle, the width of the rectangle, in terms of l, is $l - 5$. Resulting in this equation:

$$2l + 2(l - 5) = 50.$$

    Solving this equation, $l = 50$. So $\boxed{\text{the length is 15 meters, and the width is 10 meters}}$.

37. The width of a rectangle is five meters less than the length. The area of a rectangle is 176 square meters. Find the length and width of the rectangle.

    Solution: Let l be the measure of the length of the rectangle, the width of the rectangle, in terms of l, is $l - 5$. Resulting in this equation:

$$l(l - 5) = 176.$$

    Solving this equation, $l = 16$ or $l = -11$, the later being extraneous because we cannot have a negative length. So $\boxed{\text{the length is 16 meters, and the width is 11 meters}}$.

38. A steel rod 90 centimeters long is to be cut into two pieces, each to be bent to make an equilateral triangle. The length of a side of one triangle is to be twice the length of a side of the other. How should the rod be cut?

Solution: Let x be the measure of the length, in centimeters, where the rod is cut, resulting in two pieces x and $90 - x$ centimeters. Resulting in this equation:

$$2 \cdot \frac{x}{3} = \frac{90 - x}{3}$$

Solving this equation, $x = 30$. So $\boxed{\text{the 90 centimeter rod should be cut at 30 centimeters}}$.

39. A 16-foot ladder is leaning against a building. How high on the building will the ladder reach when the bottom of the ladder is 5 feet from the base of the building?

Solution: Let x be the measure, in feet, how high on the building the ladder will reach when the bottom of the ladder is 5 feet from the base of the building. Resulting in this equation:

$$16^2 = 5^2 + x^2$$

Solving this equation, $x = \pm\sqrt{231}$, but again, due to the physical nature of the problem, we can only accept the positive value. So $\boxed{\text{the } \sqrt{231} \approx 15.199 \text{ feet}}$[3] is how high the ladder will reach.

40. *Cheap Cars* rents cars for \$8.00 a day and \$0.10 for every mile driven. *Save-On Cars* rents cars for \$10.00 a day and \$0.08 per mile driven. You want to rent a car for one week. For how many miles will it cost you less to rent from *Cheap Cars* than from *Save-On Cars*?

Solution: Let x be the number of miles driven. The resulting inequality is:

$$7 \cdot 8 + 0.10x < 7 \cdot 10 + 0.08x.$$

Solving this inequality, $x < 700$. So, you have to drive $\boxed{\text{less than 700}}$ miles for *Cheap Cars* to be less than *Save-On Cars*.

41. An investment of \$3,500 is divided between two simple interest accounts. On one account, the annual simple interest rate is 5%, and on the second account the annual simple interest rate is 7.5%. How much should be invested in each account so that the total interest earned from the two accounts is \$215?

Solution: Let x be the amount of money invested at 5%, and $3500 - x$ be what remains to invest at 7.5%. The resulting equation is:

$$0.05x + 0.075\,(3500 - x) = 215.$$

Solving this equation, $x = 1900$. So, of the \$3,500 you would need to invest $\boxed{\$1{,}900 \text{ at } 5\%}$ and $\boxed{\$1{,}600 \text{ at } 7.5\%}$

[3] This is approximately 15 feet, $2\frac{3}{8}$ inches.

42. A motorboat leaves a harbor and travels at an average speed of 9 miles per hour (mph) toward a small island. Two hours later, a cabin cruiser leaves the same harbor and travels at an average speed of 18 mph toward the same island. In how many hours after the cabin cruiser leaves will the cabin cruiser be alongside the motorboat?

    Solution: Let x be the number of hours after the cabin cruiser leaves. The resulting equation is:

    $$9\,(x+2) = 18x.$$

    Solving this equation, $x = 2$. So, it will take $\boxed{\text{2 hours}}$.

43. A goldsmith combined an alloy that cost \$4.30 per ounce with an alloy that cost \$1.80 per ounce. How many ounces of each alloy were used to make a mixture of 200 ounces costing \$2.50 per ounce?

    Solution: Let x be the number of ounces of the more expensive alloy, and $200 - x$ be the number of ounces of the cheaper alloy. The resulting equation is:

    $$4.30x + 1.8\,(200 - x) = 2.50 \cdot 200.$$

    Solving this equation, $x = 56$. So you will need to use $\boxed{\text{56 ounces of the expensive alloy}}$ and $\boxed{\text{144 ounces of the cheaper alloy}}$.

44. A chemist needs to make 2 liters of an 8% acid solution by mixing a 10% acid solution and a 5% acid solution. How many liters of each solution should the chemist use?

    Solution: Let x be the number of liters of the 10% acid solution, and $2 - x$ be the number of liters of the 5% acid solution. The resulting equation is:

    $$0.10x + 0.05\,(2 - x) = 0.08 \cdot 2$$

    Solving this equation, $x = 1.2$. So you will need to use $\boxed{\text{1.2 liters of the 10\% acid solution}}$ and $\boxed{\text{0.8 liters of the 5\% acid solution}}$.

45. One grocery clerk can stock a shelf in 20 minutes, whereas a second clerk requires 30 minutes to stock the same shelf. How long would it take to stock the shelf if the two clerks worked together?

    Solution: Let t be the number of minutes that the two clerks work together on the given task. The resulting equation is:

    $$\frac{1}{20}t + \frac{1}{30}t = 1.$$

    Solving this equation, $t = 12$. So, it will take $\boxed{\text{12 minutes}}$ for the two clerks, working together, to stock the shelf.