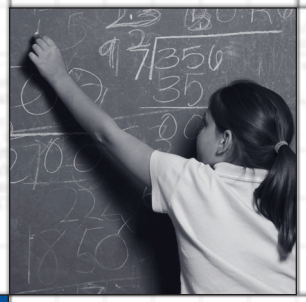
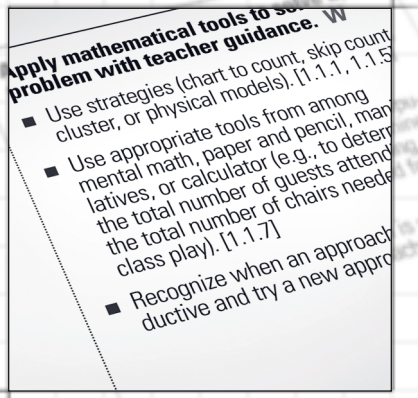
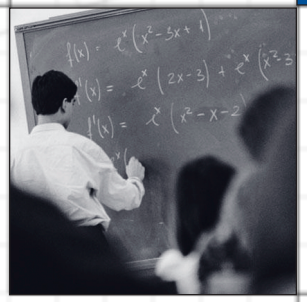


Mathematics



K–10 Grade Level Expectations:

A New Level of Specificity



Washington State's
Essential Academic Learning Requirements

To obtain copies of this document, order on-line from the Department of Printing general store: www.prt.wa.gov or call (360) 570-5024.
Please refer to OSPI Document Number 04-0006 for quicker service.

This material is available in alternative format upon request. Contact the Resource Center at (888) 595-3276 or TTY (360) 664-3631.

Contents

A New Level of Specificity

" Grade level expectations in mathematics will help more students understand and apply math concepts. This detailed roadmap outlines the mathematics skills and processes students need to develop as they progress through school. This is a much-needed, valuable tool that will help teachers guide their K–10 instruction and help all students achieve mathematics proficiency."

Dr. Terry Bergeson
Superintendent of Public Instruction

This publication is designed to help students in Washington's classrooms become proficient in the skills and processes of mathematics.

Washington's school reform efforts focus on setting clear, high expectations for what students should know and be able to do. The Essential Academic Learning Requirements (EALRs) articulate the state's expectations and learning standards. The Washington Assessment of Student Learning (WASL) measures whether students have met these standards.

The original EALRs defined benchmarks, or cumulative indicators, for grades 4, 7, and 10. Written in very broad terms to provide flexibility and local control, each district had the responsibility to determine the learning expectations for students in the other grades. Content frameworks were developed to provide grade level guidance. The new Grade Level Expectations (GLEs) provide specific learning standards for students in grades K–10, clarifying the skills and strategies all students need to demonstrate proficiency in each content area.

Just as EALRs were developed by Washington educators, administrators, parents, and community members, developing or creating the Grade Level Expectations involved hundreds of participants and countless feedback opportunities. Drafting teams not only defined what students should know and be able to do at each grade level, they developed descriptions of how students could demonstrate proficiency. The resulting "evidence of learning" statements take the specificity of the EALRs to a new level. As an example, a third grade teacher looking for signs of fluency in addition with whole numbers will expect students to describe and compare strategies to solve three-digit addition problems.

The Office of Superintendent of Public Instruction is committed to helping educators provide high quality instruction for all Washington students. This document provides all educators access to essential learning expectations to ensure all students achieve mathematics success.

A Decade of Education Reform

" ... provide students with the opportunity to become responsible citizens, to contribute to their own economic well-being and to that of their families and communities and to enjoy productive and satisfying lives."

■ **Basic Education Act Preamble**, 1993

Ten years ago, Washington established the commitment that all children would achieve at high levels. The purpose of this reform is clearly spelled out in the preamble of the Basic Education Act of 1993: *"... provide students with the opportunity to become responsible citizens, to contribute to their own economic well-being and to that of their families and communities and to enjoy productive and satisfying lives."* The law established four common learning goals for all Washington students designed to create high quality academic standards and raise student achievement. The four learning goals provided the foundation for the development of standards, called Essential Academic Learning Requirements, for reading, communications, writing, mathematics, science, social studies, health/fitness, and the arts. Establishing an assessment system to measure progress and establishing an accountability system to monitor progress complete the key components of the Basic Education Act.

Washington State Learning Goals

- **Read** with comprehension, **write** with skill, and **communicate** effectively and responsibly in a variety of ways and settings.
- **Know and apply the core concepts and principles** of mathematics; social, physical, and life sciences; civics and history; geography; arts; and health and fitness.
- **Think** analytically, logically, and creatively, and integrate experience and knowledge to form reasoned judgments and solve problems.
- **Understand** the importance of work and how performance, effort, and decisions directly affect **future career and educational opportunities**.

In the last decade, educators at every level contributed tremendous effort, bringing greater clarity to the EALRs. The creation of Grade Level Expectations is a logical next step to provide educators with greater specificity, as well as to respond to the Elementary and Secondary Act of 2001. This federal legislation, known as the *No Child Left Behind Act*, calls for each state to adopt challenging academic standards for all students. The Grade Level Expectations will be used to develop new assessments in reading, mathematics, and science required by this law.

Mathematics EALRs with Grade Level Expectations

K–10 EALR Statement

K–10 Component

Grade Level Expectations (GLEs)

K	1	2	3	4	5	6	7	8	9/10

- Evidence of Learning
- Evidence of Learning
- Evidence of Learning

EALR 1: The student understands and applies the concepts and procedures of mathematics.

EALR 1 is commonly referred to as the content (or content strands) of mathematics. This EALR is subdivided into five components: *number sense, measurement, geometric sense, probability and statistics,* and *algebraic sense*.

EALR 2: The student uses mathematics to define and solve problems.

Problem solving should be "… a primary goal of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned" (*Curriculum and Evaluation Standards for School Mathematics, NCTM 1989*).

EALR 3: The student uses mathematical reasoning.

A major goal of mathematics instruction is to help children believe they can do mathematics and have control over their own success. Autonomy develops as children gain confidence in their ability to reason and justify their thinking. This power grows as children learn that mathematics is not simply memorizing rules and procedures, but also using logic to develop understanding (*ibid.*).

EALR 4: The student communicates knowledge and understanding in both everyday and mathematical language.

Mathematics is a language and science of patterns and means for describing the world in which we live. With its symbols and vocabulary, mathematics offers a universal way of communicating about relationships and patterns.

EALR 5: The student understands how mathematical ideas connect within mathematics, or other subject areas, and to real-life situations.

It is important that children see how mathematical ideas are related and connect ideas among and within areas of mathematics. Without such connections, children have to learn and remember isolated concepts and skills rather than overarching principles. When mathematical ideas are connected to everyday experiences, both in and out of school, children learn to value and appreciate the usefulness of mathematics (*ibid.*).

Grounding the Work in Research

Research documents utilized to develop the GLEs were the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989), *Principles and Standards* (NCTM 2000), *Adding It Up: Helping Children Learn Mathematics* (NRC 2001), *On the Shoulders of Giants: New Approaches to Numeracy* (MSEB 1990), *A Research Companion to Principles and Standards for School Mathematics* (NCTM 2003), and *Elementary and Middle School Mathematics: Teaching Developmentally* (Van de Walle 2003), *Culturally Responsive Teaching: Theory, Research, & Practice* (Dr. Geneva Gay 2000).

Technology

Technology should be available and used throughout the K–12 mathematics curriculum. In the early years, students can use basic calculators to examine and create patterns of numbers. In the upper elementary and lower middle school years, students should be encouraged to continue examining patterns, and also extend the use of technology to create charts and graphs and to develop reports. Students at these ages can also use spreadsheet software to enhance their algebraic understanding of variables and iteration. During the middle school and high school years, a wide range of technology should be available to examine complex numerical ideas, data, functions and their graphs, and interactive systems. Technology should also be used to create and examine geometric relationships, as well as to communicate.

Culturally Responsive Teaching

For all students to meet grade level expectations, mathematics instruction should "… incorporate everyday-life concepts, such as economics, employment, consumer habits, of various ethnic groups. In order to teach to the different learning styles of students, activities should reflect a variety of sensory opportunities — visual, auditory, tactile." (Gay, 2000)

Culturally responsive teaching defines the context of the mathematics classroom and may well provide the cornerstone that allows all of our students to achieve proficiency in mathematics. Culturally responsive teaching:

- Acknowledges the legitimacy of the cultural heritages of different ethnic groups, both as legacies that affect students' dispositions, attitudes, and approaches to learning and as worthy content to be taught in the formal curriculum;
- Builds bridges of meaningfulness between home and school experiences as well as between academic abstractions and lived socio-cultural realities;
- Uses a wide variety of instructional strategies that are connected to different learning styles;
- Teaches students to know and praise their own and each others' cultural heritages; and
- Incorporates multicultural information, resources, and materials in all the subjects and skills routinely taught in schools (*ibid.*, p. 29).

Understanding Grade Level Expectations

An ***Essential Academic Learning Requirement*** is a broad statement of learning that applies to grades K–10.

The ***Component*** is a K–10 statement that further defines the EALR. There is at least one component for each EALR.

The ***Grade Level Expectation*** is a statement of *cognitive demand,* using Bloom's Taxonomy, and the *essential content* or *process to be learned.* The statement, specific to one or more grades, defines the component.

The ***Evidence of Learning*** is a bulleted list of *student demonstrations* that provide educators with common illustrations of the learning. Because the bulleted list is not exhaustive, educators are encouraged to seek additional evidence of student learning.

The GLE ***Numbering System*** identifies the EALR, the component, and the GLE. For example, in the number 1.2.1, the first number stands for the EALR, the second for the component, and the third for the GLE. Note: Grade levels are not referenced in the numbering system.

Grade Level Expectations with a "**w**" denote the specific expectations which are eligible for the WASL. Not all GLEs have a "**w**." Note: Narrowing instruction to just those expectations with a "**w**" may adversely affect student mathematics success.

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.2: Understand and apply concepts and procedures from measurement.

GLE	Grade 5
Attributes, units, and systems	
1.2.1	**Understand the concept of angle measurement.** W ■ Describe and compare angles in a variety of objects. [CU] ■ Identify angles in the environment. [MC] ■ Classify or sort angles as right, acute, or obtuse. [RL, CU] ■ Identify types of angles in polygons (e.g., right, acute, obtuse). [MC] ■ Explain and provide examples of how angles are formed.

Connecting Content and Process

The relationship between content and process in mathematics is critical. It is the combination of these that gives students mathematical power. Either used in isolation will not develop mathematically proficient students. Teachers are expected to use instructional practices that provide opportunities for students to experience both on a regular basis.

Links between content and process are noted throughout the document. GLEs from EALR 1 (commonly referred to as the *content strands*) include references to the mathematical processes. GLEs for the mathematical processes (EALRs 2–5) include references to content GLEs from EALR 1. These references are found in brackets following evidence of learning statements.

Content to Process Example:

Grade 5

GLE 1.2.1: Understand the concept of angle measurement.

- Identify types of angles in polygons (e.g., right, acute, obtuse). **[MC]**

The **[MC]** links the geometric concept of angle measurement (content) to the mathematical process of Makes Connections.

EALR 2 — [SP] Solves Problems
EALR 3 — [RL] Reasons Logically
EALR 4 — [CU] Communicates Understanding
EALR 5 — [MC] Makes Connections

Process to Content Example:

Grade 3

GLE 4.1.2: Understand how to extract information for a given purpose from one or two different sources using reading, listening, and observation.

- Read and report on data from tables, charts, and bar graphs. **[1.4.5]**

The **[1.4.5]** refers to the GLE from EALR 1 in Grade 3. This links the process of Communicates Understanding (EALR 4) to content in probability and statistics.

Alignment for Student Achievement

" Without alignment, there can be no fair judgment about how well schools are really doing."

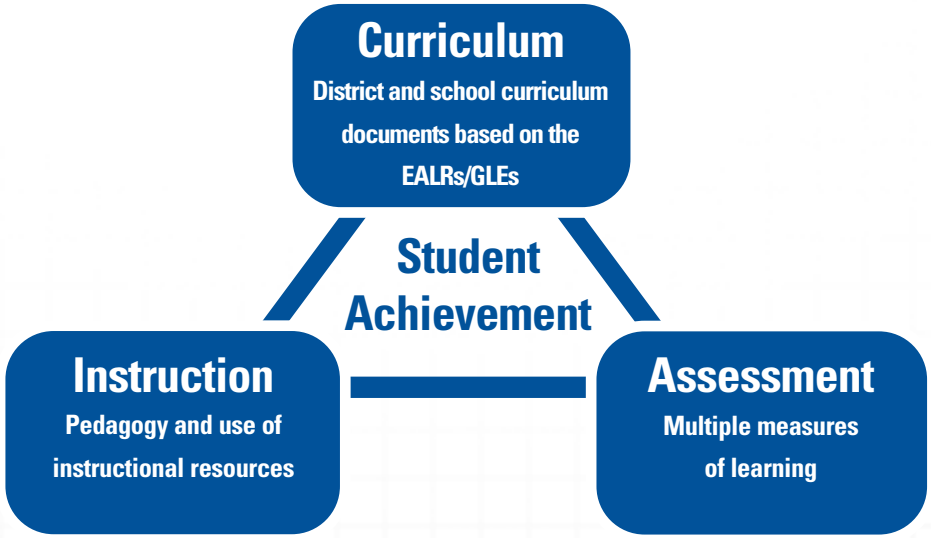
■ **Fenwick English**, 2000

It is critical that the **curriculum**, **instruction**, and **assessment** deeply align. The Essential Academic Learning Requirements (EALRs), including the Grade Level Expectations (GLEs), provide the foundation for the development of district and school curriculum documents. Instruction refers to both the teacher's pedagogy and use of instructional resources. Assessment includes diagnostic, formative (classroom-based), and summative (including the WASL) assessments.

Alignment refers to the match between curriculum, instruction, and assessment in regard to the **content**, the **context**, and the **cognitive demand** of the learning. The content identifies the concepts, procedures, and/or processes to be learned. **Topical alignment** occurs when content and curriculum match.

Context encompasses the learning environment, format, instructional resources and support provided students for acquiring and practicing the content. Cognitive demand refers to the type of cognition required of the student, as defined in Bloom's Taxonomy of the Cognitive Domain (see appendix). It is important to note that the use of Bloom's Taxonomy in this document reflects a classification of six types of cognition rather than a hierarchy of dependent levels of cognition.

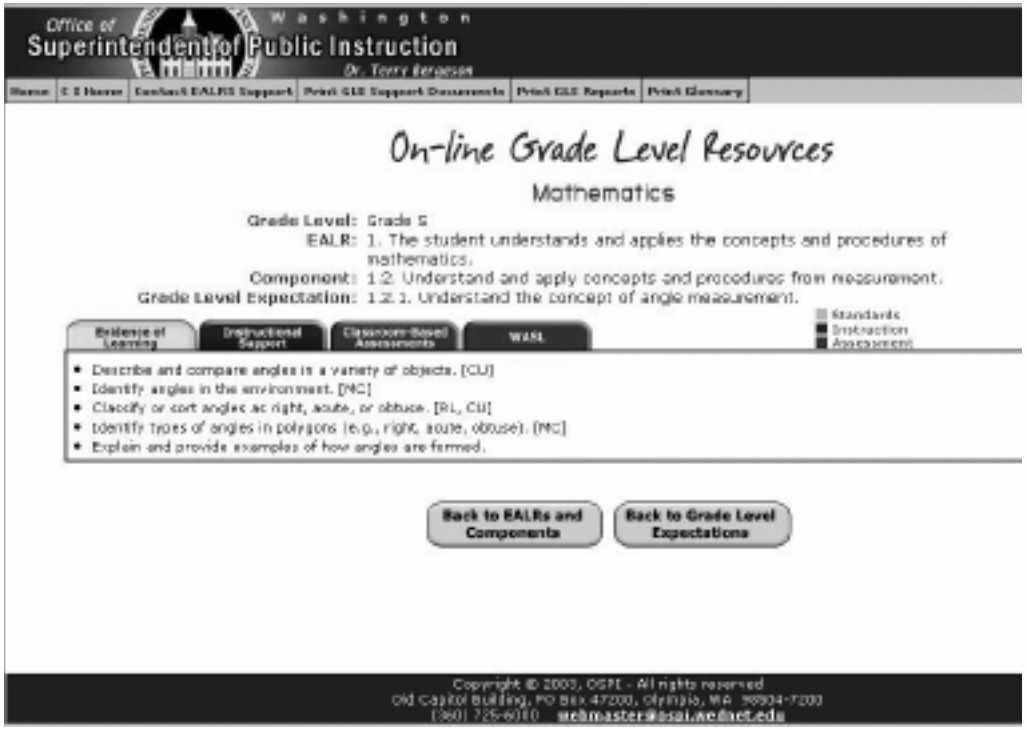
Deep alignment requires not only **content** alignment, but alignment of **context** and **cognitive demand** as well.



Accessing the On-line Grade Level Resources

Aligned GLE support can be accessed via On-line Grade Level Resources at the Curriculum and Instruction home page on the OSPI website. This interactive resource provides the following features:

- GLE Reports (grade level, grade spans, K–10 GLEs).
- Links to GLE glossary.
- Aligned instructional support.
- Integration links to other content areas.
- Support for classroom-based assessments.
- Links to WASL strands, learning targets, released items, and annotations.



www.k12.wa.us/CurriculumInstruct

An Overview of K–10 Mathematics Instruction

The Grade Level Expectations (GLEs) describe a connected series of learning competencies necessary to create mathematically proficient citizens. The GLEs define the knowledge and skills that students should gain from kindergarten through the 10th grade. These expectations should not be the end of the mathematical experience for students. Rather, they serve as a solid foundation on which continued application and learning of mathematics contributes to success in high school and beyond.

Kindergarten

In kindergarten, students begin developing the concept of number by counting, representing and ordering, combining, sorting, and comparing sets of objects. They understand addition as putting sets together. In describing and identifying objects based on attributes and recognizing and describing simple repeating patterns, students develop a beginning sense of geometry and algebra. They also develop an understanding of the relationship between data and picture representations of the data.

Grade 1

In first grade, students count, sort, and compare sets, understanding the relative values of numbers. Students understand subtraction as separating or undoing addition and expand their understanding of number through application of basic addition and subtraction facts. Students read a clock, work with two-dimensional figures and use nonstandard units to measure. They also develop their understanding of statistics by organizing and interpreting data. They recognize and describe simple repeating and growing patterns to develop their algebraic sense.

Grade 2

In second grade, students expand their understanding of number to include three-digit numbers. They continue to gain proficiency in the basic addition and subtraction facts and expand concepts in measurement, using procedures to find measures (time, weight). By interpreting and creating picture and bar graphs, students further develop their early understanding of statistics. Students also work with a variety of patterns and use symbols to describe numerical relationships.

Grade 3

In third grade, students develop their fluency with addition and subtraction, while beginning to understand multiplication and division as repeated addition and subtraction, respectively. Students use standard units of measure for temperature, length, liquid volume, and weight. Students gain a broader understanding of geometry by identifying properties of shapes and line segments. Algebraic sense grows through their understanding of equality and by identifying missing numbers in addition and subtraction expressions and equations.

Grade 4

In fourth grade, students become proficient with multiplication and division of whole numbers, while developing an understanding of fractions and decimals. In measurement, they develop an understanding of area. The concept of probability as chance is developed and fourth graders continue to expand their understanding of statistics using graphing and measures of central tendency. Students refine their estimation skills for computation and measurement and develop an understanding of the relationships between and among two-dimensional (plane) figures. They graph points in the first quadrant on a coordinate plane and extend and duplicate patterns. Students recognize a geometric transformation, such as a reflection (flip) and a translation (slide).

Grade 5

In fifth grade, students become proficient using non-negative rational numbers to solve problems. They apply procedures to measure a variety of geometric figures and collect, display, and analyze data. Students examine the basis of probability, and also the mean. They solve problems involving area and perimeter and further develop algebraic sense through variable expressions and open sentences.

Grade 6

In sixth grade, students begin developing their understanding of negative numbers with the introduction of integers. Students also begin working with other representations of rational numbers. They examine the concept of volume, as well as collect, analyze, display, and interpret data, using a variety of graphical and statistical methods. They find the probability of events and analyze numerical and geometric patterns. Students also develop an understanding of algebraic terms and solve algebraic equations in one variable.

Grade 7

In seventh grade, students complete their development of the rational number system with the inclusion of negative decimals and fractions. Fluency of all operations on non-negative rational numbers is expected of students, as is proficiency with addition and subtraction of all rational numbers. Students understand proportional reasoning and similarity and use these concepts to solve problems. They locate points in any of the four quadrants on a grid and translate linear relationships in table, graph and equation forms. Students extend their understanding of probability into multiple events. Algebraic sense also develops as students solve two-step equations in one variable.

Grade 8

In eighth grade, students are proficient in computation with all rational numbers and use proportions to solve a variety of problems. They understand the need for precision when measuring and use derived units of measure. Students understand the concept of distance and the relationship between distance and the Pythagorean Theorem. They recognize three-dimensional shapes represented in two-dimensional drawings and apply transformations to geometric shapes in the coordinate plane. Eighth graders find probability of compound events and analyze bivariate data sets. They also understand recursive forms of linear and exponential relationships and solve two-step equations and inequalities.

Grades 9–10

In ninth and tenth grades, there may be a number of different course offerings for students. Regardless of the particular title of the course, students will be proficient with operations on rational numbers in all forms and scientific notation representing very large and very small numbers. Students analyze effects of changes in dimension and apply formulas to measurement. They understand both the U.S. and metric systems and are able to convert units within each system. Students use a variety of methods and formulas to find area, volume, the slope of a line, and the distance between points on a coordinate grid. They apply multiple transformations to figures or points, and can apply conditional probability in situations. Students develop equations for linear models. They analyze statistical arguments for accuracy and bias, develop arithmetic and geometric patterns using recursive definitions, and solve multi-step equations and systems of equations in two variables.

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.1: Understand and apply concepts and procedures from number sense.

GLE	K	1	2	3	4
Number and numeration					
1.1.1	**Understand the concept of number.** ■ Count to at least 31. ■ Represent a number to at least 10 in different ways (e.g., numerals, spoken words, pictures, physical models). [CU] ■ Show that the last count word names the quantity of the set (cardinality) (i.e., when counting fingers on a hand "one, two, three, four, five," the "five" says how many fingers there are). [CU, MC] ■ Identify the base ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. ■ Explain how numbers are used and give examples (e.g., to count, to order). [CU]	**Understand different representations of whole numbers.** ■ Represent a number to at least 100 in different ways (e.g., numerals, pictures, words, physical models) and translate from one representation to another. [CU] ■ Group and regroup objects into 1s and 10s. ■ Count sets of objects less than 100 using a variety of grouping strategies.	**Understand place value in whole numbers.** ■ Group and regroup objects into 1s, 10s, and 100s and explain relationships. [CU] ■ Determine the value of a digit based on its position in a number. ■ Read and write numbers to at least 1,000. [CU]	**Understand the concept of whole numbers.** W ■ Represent a number to at least 10,000 in different ways (e.g., words, numerals, pictures, physical models). [CU] ■ Translate from one representation of a whole number to another in standard, expanded, and word forms. [MC] ■ Generate equivalent representations for a given number by decomposing and composing. [MC] ■ Explain the difference between the natural numbers and the whole numbers. ■ Identify place values of digits of whole number to the hundreds or thousands place using words, pictures, or numbers. ■ Write whole numbers to 999. ■ Decompose whole numbers into components (e.g., 35 is made of 3 tens and 5 ones) using words, numbers, or pictures.	**Understand the concept of decimals (money) and fractions.** W ■ Interpret fractions as parts of a whole object, number, or set (e.g., half of a medium pizza and half of a large pizza are not equal amounts). ■ Symbolically represent parts of a whole or parts of a set with common denominators. [CU] ■ Explain how fractions (denominators of 2, 3, 4, 6, and 8) represent information across the curriculum (e.g., interpreting circle graphs, fraction of states that border an ocean). [CU, MC] ■ Represent decimals (money) in multiple ways (e.g., symbols, physical models). [CU] ■ Explain or show how a fraction can be decomposed into smaller fractions (e.g., $^3/_4 = {}^1/_4 + {}^1/_4 + {}^1/_4$).
1.1.2	**Understand sequential relationships among whole numbers.** ■ Tell what number comes before or after a given number. ■ Use comparative language (e.g., less than, more than, equal to) to compare numbers to at least 20. [CU] ■ Use a known quantity to at least 10 (benchmark) to compare sets (e.g., sets of counters). ■ Identify the ordinal position of objects at least through tenth (e.g., first, second . . .).	**Understand sequential relationships among whole numbers.** ■ Order three or more numbers to at least 100 from smallest to largest. [RL] ■ Use comparative language (e.g., less than, more than, equal to) to compare numbers to at least 100. [CU] ■ Skip count by 2, 5, and 10. ■ Count forward and backward, from a given number that is less than 100.	**Understand sequential relationships among whole numbers.** ■ Order three or more numbers to at least 1,000 from smallest to largest. [RL] ■ Use comparative language (e.g., less than, more than, equal to) to compare numbers to at least 1,000. [CU]	**Understand the relative values of whole numbers.** W ■ Compare whole number values to at least 10,000 using the symbols for "greater than," "less than," and "equal to." ■ Order three or more numbers to at least 10,000 from smallest to largest. [CU] ■ Compare combined quantities (e.g., 50 + 3 is greater than 40 + 9). [RL]	**Understand the relative values of fractions and decimals (money).** W ■ Model and describe equivalent fractions (e.g., paper folding, geoboards, parallel number lines). [CU] ■ Use a number line to approximate and label halves, thirds, and fourths in relationship to whole units. [CU, MC] ■ Order fractions with like denominators. [RL] ■ Demonstrate and explain equivalent relationships between decimals and fractions (e.g., $.50 is equal to 1/2 a dollar and 50/100 of a dollar) using models. [CU, MC] ■ Demonstrate or show the order of like-denominator fractions using pictures or objects. [CU]

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

GLE	5	6	7	8	9/10
Number and numeration					
1.1.1	**Understand the concepts of fractions and decimals.** W ■ Represent mixed numbers, improper fractions, and decimals. ■ Create a model when given a symbolic representation or write the fraction when given a model (e.g., number line). [CU] ■ Explain the value of a given digit in a decimal to at least the thousandths place. [CU] ■ Explain how the value of a fraction changes in relationship to the size of the whole (e.g., half a pizza vs. half a cookie). [CU] ■ Use factors and multiples to rename equivalent fractions. [RL] ■ Read and write decimals to at least the thousandth place. [CU] ■ Demonstrate and explain equivalent relationships between decimals and fractions (e.g., $.50 is equal to ½ a dollar and 50/100 of a dollar) using models. [CU, MC] ■ Convert between improper fractions and mixed numbers. [MC]	**Understand the concept of integers as the set of natural numbers (1, 2, 3 …), their opposites (-1, -2, -3 …), and 0.** W ■ Illustrate integer values using models and pictures (e.g., temperature, elevators, net worth/debt, riding a bus or subway). [CU] ■ Apply rules of divisibility to show if a quotient is an integer. [RL] ■ Explain the meaning of integers and give examples. ■ Identify the opposite of a given integer.	**Understand the concept of rational numbers (integers, decimals, fractions).** W ■ Create a model when given a symbolic representation of a rational number. [CU, MC] ■ Write the rational number when given a model (e.g., number line, area model, situation, diagram, picture). [CU, MC] ■ Identify and convert between equivalent forms of rational numbers (e.g., fractions to decimals, percents to fractions). [MC] ■ Identify prime, square, or composite numbers. [CU] ■ Explain the meaning of rational numbers and give examples. [CU]	**Understand the concept of rational numbers including whole number powers and square roots of square numbers.** W ■ Explain the meaning of a whole number exponent. [CU] ■ Read and use exponential notation to represent large numbers (e.g., $2500 = 50^2$). [MC] ■ Identify a square number and find its root. ■ Identify different representations of rational numbers and select the best representation in the situation (e.g., percent for sales discount or sales tax, fraction for probability, and decimals for money, distance [4.35 kilometers], batting averages). ■ Write a squared number.	**Understand and apply scientific notation.** W ■ Read and use scientific and exponential notation. [MC, RL] ■ Identify a real-life situation to match a particular number written in scientific or exponential notation and justify the answer. [MC, RL] ■ Use scientific or exponential notation to simplify a problem. [RL, MC] ■ Illustrate the meaning of scientific notation using pictures, diagrams, or numbers. [CU] ■ Read and translate numbers represented in scientific notation from calculators and other technology, tables, and charts.
1.1.2	**Understand the relative values of non-negative fractions or decimals.** W ■ Compare, order, or illustrate whole numbers, decimals, and fractions (denominators of 2, 3, 4, 5, 6, or 10) using concrete models (e.g., number line or shaded grid) or implementing strategies (e.g., like denominators, benchmarks, conversions). [RL, CU] ■ Determine equivalence among fractions. [RL] ■ Explain why one fraction is greater than, equal to, or less than another fraction. [CU] ■ Explain why one decimal number is greater than, equal to, or less than another decimal number. [CU]	**Understand the relative values of integers and non-negative rational numbers.** W ■ Compare different representations of non-negative rational numbers by implementing strategies (e.g., like denominators, changing to the same form). [RL, CU, MC] ■ Identify equivalence between non-negative integers, fractions, percents, and decimals. [MC] ■ Compare and order integer values and explain which is greater and why (e.g., place the integers on a number line). [CU] ■ Represent and identify integers on a model (e.g., number line, fraction line, or decimal grid). [RL, CU]	**Understand the relative values of rational numbers.** W ■ Compare and order rational numbers using physical models or implementing strategies (e.g., like denominators, changing to the same form). [RL, MC] ■ Locate symbolic representations of rational numbers on a model (e.g., a number line, fraction line, decimal grid, and circle graph). [MC] ■ Explain the value of a given digit in a rational number (e.g., 2.3 is 2 ones and 3 tenths). [CU]	**Understand the relative values of rational numbers including whole number powers and square roots of square numbers.** W ■ Compare and order rational numbers using models or implementing strategies. [RL] ■ Order different representations of rational numbers. [RL] ■ Place symbolic representations of rational numbers on a number line including whole number powers and square roots of square numbers. [CU]	

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.1: Understand and apply concepts and procedures from number sense.

GLE	K	1	2	3	4
Number and numeration					
1.1.3				**Understand and apply the commutative and identity properties of addition on whole numbers.** W ■ Explain or show how the commutative property works with addition and not subtraction using words, numbers, or physical models. [CU] ■ Describe how the identity property works with addition. [CU] ■ Determine whether addition equations are true or false and explain, based on the commutative or identity properties for addition (e.g., 15 + 3 + 5 = 15 + 5 +3). [CU] ■ Identify an equivalent expression using the commutative property. ■ Show how the commutative property works using pictures or objects. [CU]	**Understand and apply the associative property of addition and multiplication and the commutative, identity, and zero properties of multiplication on whole numbers.** W ■ Describe how the commutative property works with multiplication and not division using words, numbers, or physical models. [CU] ■ Describe how the identity property for addition is different from the identity property for multiplication using words, numbers, pictures, or physical models. [CU] ■ Determine whether equations are true or false and explain, based on any of the properties for multiplication (e.g., 4 x (5 x 6) = (4 x 5) x 6). [CU] ■ Determine whether equations are true or false and explain, based on any of the properties (e.g., 14 + (62 + 38) = (14 + 62) + 38). [CU] ■ Demonstrate commutative, associative, or identity properties of addition or multiplication using pictures or objects. [CU]
1.1.4					

GLE	5	6	7	8	9/10
Number and numeration					
1.1.3	**Understand and apply the concept of divisibility.** W ■ Apply the concepts of odd and even numbers to check for divisibility, finding factors and multiples. ■ Illustrate prime or composite numbers by creating a physical model (e.g., arrays, area models). [CU] ■ Identify the prime numbers between 1 and 100. ■ Explain why a whole number between 1 and 100 is prime or composite. [CU] ■ Explain a method to find the least common multiple (LCM) and greatest common factor (GCF) of two numbers. [CU] ■ Solve problems related to primes, factors, multiples, and composites in a variety of situations (e.g., find a mystery number, find unit pricing, increase or decrease a recipe, find the portions for a group). [SP] ■ Factor a number into its prime factors. ■ Determine whether one number is a factor of another number.	**Apply properties of addition and multiplication to non-negative rational numbers.** W ■ Illustrate and explain the commutative and associative properties and why they work (e.g., use physical models, pictures). [CU] ■ Use addition and multiplication properties to assist in computations (e.g., 5 • 7 • 6 can be rewritten as 5 • 6 • 7, which is 30 • 7 or 210). ■ Determine whether a solution is accurate based on application of commutative, associative, and identity properties of addition and/or multiplication. [RL]	**Apply properties of addition and multiplication including inverse properties to the rational number system.** W ■ Use the inverse relationships between multiplication and division to simplify computations and solve problems. [SP, RL] ■ Use the inverse properties of addition and multiplication to simplify computations with integers, fractions, and decimals. [SP, RL, MC] ■ Identify the inverse elements when using the additive inverse and the multiplicative inverse properties (e.g., 8 + -8 = 0; 2 x $^1/_2$ = 1). ■ Use the additive inverse property to solve problems. [RL] ■ Illustrate or explain the additive and multiplicative inverse properties and why they work. [CU]	**Apply properties of addition, multiplication, and the distributive property to the rational number system.** W ■ Illustrate and explain the distributive property of multiplication over addition (e.g., using an area model or picture). [CU] ■ Use the distributive property to simplify expressions including those using integers. [RL] ■ Use the distributive property to factor expressions (e.g., 3 • 9 + 3 = 3 • (9+1)). [RL] ■ Identify the multiplicative inverse of a number.	
1.1.4		**Understand the concepts of ratio and percent.** W ■ Write ratios in part/part and part/whole relationships using objects, pictures, and symbols (e.g., using /, :, or "to" as representations for ratios). [CU] ■ Represent equivalent ratios using objects, pictures, or symbols. [CU] ■ Represent equivalent percentages using objects, pictures, and symbols. [CU] ■ Identify percent as 100 equal-size parts of a set (e.g., 1% of 200 items is 2 items). ■ Explain ratio and percents and give examples of each. [CU]	**Understand the concept of direct proportion.** W ■ Express proportional relationships using objects, pictures, and symbols. [CU] ■ Explain the meaning of a proportion. [CU] ■ Represent a new relationship from a given ratio (e.g., height of a totem pole, maypole). [MC] ■ Represent percentages less than 1% or greater than 100% using objects, pictures, and symbols. [CU] ■ Complete or write a proportion for a given situation. [CU] ■ Solve problems involving proportions (e.g., determine the number and kinds of baked goods to bring to a bake sale based on proportions of different goods sold at previous bake sales). [SP, MC] ■ Use ratios to make predictions about proportions in a future situation. [RL, MC]	**Apply ratio, percent, and direct proportion in situations.** W ■ Solve problems involving ratio and proportion (e.g., similar figures, scale drawings, rates, find unit pricing, increase or decrease a recipe, find the portions for a group converting between different units of measure, or finding medicinal dosages). [SP, MC] ■ Solve problems involving percentages (e.g., percent increase/decrease, tax, commission, discount). [SP, MC] ■ Explain advantages and disadvantages of different representations of ratios or percents in a given situation (e.g., using $^1/_8$ versus 12 $^1/_2$%). [CU, MC] ■ Determine an unknown value for a dimension or a number of events or objects using ratio or proportion. ■ Complete a proportion in a situation.	**Apply understanding of direct and inverse proportion to solve problems.** W ■ Explain a method for determining whether a real-world problem involves direct proportion or inverse proportion. [SP, CU, MC] ■ Explain a method for solving a real-world problem involving direct proportion. [CU, MC] ■ Explain a method for solving a real-world problem involving inverse proportion. [CU, MC] ■ Solve problems using direct or inverse models (e.g., similarity, age of car vs. worth). [SP, MC] ■ Explain, illustrate, or describe examples of direct proportion. [CU] ■ Explain, illustrate, or describe examples of inverse proportion. [CU] ■ Use direct or inverse proportion to determine a number of objects or a measurement in a given situation.

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.1: Understand and apply concepts and procedures from number sense.

GLE	K	1	2	3	4
Computation					
1.1.5	**Understand the meaning of addition.** ■ Express stories involving addition (e.g., join) with models, pictures, and symbols. [CU, MC] ■ Use addition in the classroom environment (e.g., tables and chairs in the classroom). [MC]	**Understand the meaning of subtraction.** ■ Express stories involving subtraction (e.g., separate) with models, pictures, and symbols. [CU, MC] ■ Show relationships between addition and subtraction using physical models, diagrams, and acting out problems. [CU]	**Understand the meaning of addition and subtraction and how they relate to one another.** ■ Show relationships between addition and subtraction using physical models, diagrams, and acting out problems. [CU, MC] ■ Model real-life situations involving addition (e.g., Peter has 7 peanut butter cookies and 4 chocolate chip. How many cookies does he have?) and subtraction (e.g., Peter has 11 cookies which is 4 more than Teresa. How many cookies does Teresa have?) using physical models and diagrams from various cultures and acting out problems. [CU]	**Understand the meaning of multiplication and division on whole numbers.** W ■ Illustrate multiplication and division using models and diagrams. [CU] ■ Illustrate and explain the inverse relationship between multiplication and division using physical diagrams, words, and symbols (e.g., arrays, fact families). [CU] ■ Describe and compare strategies to solve problems involving multiplication and division (e.g., alternative algorithms, different strategies, decomposition, properties of multiplication). [CU] ■ Demonstrate the relationship between multiplication and repeated addition. ■ Demonstrate the relationship between division and repeated subtraction.	**Understand the meaning of addition and subtraction on like-denominator fractions.** W ■ Represent addition and subtraction of fractions with like denominators using models (e.g., everyday objects, fraction circles, number lines, geoboards). [CU] ■ Explain the meaning of addition and subtraction of like-denominator fractions. [CU] ■ Represent addition or subtraction of like-denominator fractions that represent sets of objects (e.g., $\frac{1}{4}$ of 24 marbles plus $\frac{1}{4}$ of 24 marbles = $\frac{2}{4}$ of 24 marbles or 12). ■ Demonstrate the meaning of addition or subtraction of like denominators with multiple examples. [CU]
1.1.6		**Understand and apply procedures for addition of whole numbers with fluency.** ■ Use strategies (e.g., count on, count back, doubles) for addition to at least sums to 12. [SP, RL] ■ Recall addition facts through at least sums to 12. ■ Solve problems involving addition using procedures and explaining those procedures. [SP, RL, CU]	**Understand and apply procedures for addition and subtraction of whole numbers with fluency.** ■ Use strategies for addition and subtraction combinations through at least 18. ■ Recall addition and subtraction facts through at least 18. ■ Solve problems involving addition and subtraction with two- or three-digit numbers using a calculator and explaining procedures used. [SP, CU] ■ Make combinations and name total value of coins.	**Apply procedures of addition and subtraction on whole numbers with fluency.** W ■ Describe and compare strategies to solve three-digit addition and subtraction problems (e.g., child-developed algorithms, decomposition). [RL, CU] ■ Use joining, separating, adding-on, and finding the difference to add and subtract. ■ Write and solve multi-step problem situations that involve addition and subtraction. [CU, MC] ■ Use calculators to compute with large numbers (e.g., adding three or more three-digit numbers; subtracting three digit from four digit numbers).	**Apply procedures of multiplication and division on whole numbers with fluency.** W ■ Use a variety of strategies to mentally access multiplication and division facts through 12s. ■ Recall multiplication and division facts through 12s. ■ Record, share, and evaluate algorithms used in computational situations. [CU] ■ Write and solve problem situations with whole numbers using a combination of any two operations. [CU, MC] ■ Interpret remainders of a division problem in a given situation. [RL, MC] ■ Use calculators to compute with large numbers (e.g., multiplying two digits times three digits; dividing three or four digits by two digits without remainders.

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

GLE	5	6	7	8	9/10
Computation					
1.1.5	**Understand the meaning of addition and subtraction on non-negative decimals and fractions.** W ■ Explain the meaning of adding and subtracting fractions and decimals using words, symbols, or other models (e.g., fractions with denominators of 2, 4, 8 or 2, 3, 6, 12 or 5, 10 — highest LCM of 12). [CU] ■ Create a problem situation involving addition or subtraction of non-negative decimals or fractions. [SP, RL, CU, MC] ■ Represent addition and subtraction of decimals through hundredths using models (e.g., with money). [CU] ■ Create or identify a representation of addition or subtraction of non-negative decimals or fractions. ■ Demonstrate the effect of multiplying a whole number by a decimal number. [CU]	**Understand the meaning of multiplication and division on non-negative rational numbers.** W ■ Explain the meaning of multiplying and dividing non-negative fractions and decimals using words or visual or physical models (e.g., sharing a restaurant bill, cutting a board into equal-sized pieces, drawing a picture of an equation or situation). [CU, MC] ■ Explain why multiplication of fractions can be done by multiplying denominators while addition of fractions requires finding common denominators. [CU] ■ Use technology to demonstrate how multiplication and division with decimals affects place value.	**Understand the meaning of addition and subtraction on integers.** W ■ Explain the meaning of addition and subtraction of integers using real-world models (e.g., reducing debt, temperature increase or decrease, yards gained and lost, movement of a hot-air balloon). [CU, MC] ■ Create a problem situation involving addition or subtraction of integers. [CU, MC] ■ Explain or show the meaning of addition or subtraction of integers. [CU] ■ Use technology to demonstrate addition and subtraction with integers.	**Understand the meaning of operations on rational numbers (including square roots of square numbers and whole number powers).** W ■ Create a problem situation to match a given rational number equation. [CU, MC] ■ Explain the meaning of negative and zero exponents. [CU] ■ Demonstrate or describe the meaning of multiplication and division of integers using words, visual, or physical models. [CU] ■ Create a problem situation involving multiplication or division of integers. [CU, MC] ■ Explain solutions when dividing by fractions (e.g., when dividing by a number between 0 and 1, the result is larger than the dividend). [CU]	
1.1.6	**Apply procedures of addition and subtraction with fluency on non-negative decimals and like-denominator fractions.** W ■ Add and subtract like-denominator fractions (denominators of 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 16, 20, and 100) and non-negative decimals. ■ Explain a strategy for adding fractions. [CU] ■ Write and solve problem situations to find sums or differences of decimals or like-denominator fractions. [CU, MC] ■ Use calculators to multiply or divide with two decimal numbers in the hundredths and/or thousandths place.	**Apply computational procedures with fluency for addition and subtraction on non-negative rational numbers.** W ■ Find the sums or differences of non-negative fractions or decimals. ■ Write and solve real-world problem situations to find sums or differences of decimals or fractions. [CU, MC] ■ Use the least common multiple and the greatest common factor of whole numbers to solve problems with fractions (e.g., to find a common denominator, to add two fractions, or to find the simplified form for a fraction). [MC] ■ Use addition and subtraction to solve real-world problems involving non-negative rational numbers. [SP] ■ Solve multiple-step computations requiring one, two, or more different operations. [MC]	**Apply computational procedures with fluency for multiplication and division on non-negative rational numbers.** W ■ Find the product or quotient using non-negative decimals and fractions with unlike denominators. ■ Apply percentages to solve a problem in a variety of situations (e.g., taxes, discounts, interest). [SP, MC] ■ Use multiplication and division to solve real-world problems involving non-negative rational numbers. [SP] ■ Multiply non-negative decimal numbers to the hundredths place. ■ Divide non-negative decimals numbers to the thousandths place by non-negative decimal numbers to the hundredths place.	**Apply computational procedures with fluency on rational numbers including whole number powers and square roots of square numbers.** W ■ Compute with rational numbers using order of operations. ■ Compute fluently with rational numbers in all forms except exponential. ■ Write and solve problems that involve computation with rational numbers. [CU, MC] ■ Solve problems using rational numbers with whole number powers. [SR] ■ Solve problems using rational numbers with square roots of perfect squares (e.g., given a square garden with an area of nine square meters, how much fence would be needed to encompass a garden twice the size of the original garden). [SR]	**Apply strategies to compute fluently with rational numbers in all forms including whole number exponents.** W ■ Complete multi-step computations using order of operations in situations involving combinations of rational numbers including whole number exponents and square roots of square numbers. [MC] ■ Calculate using order of operations on all forms of rational numbers (e.g., $(3 \bullet 2+5)^2 - 8$, $2^2 + 3^2$). ■ Use properties to reorder and rearrange expressions to compute more efficiently. [RL]

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.1: Understand and apply concepts and procedures from number sense.

GLE	K	1	2	3	4
Computation					
1.1.7		**Understand and apply strategies and appropriate tools for adding with whole numbers.** ■ Use strategies and appropriate tools from among mental math, paper and pencil, manipulatives, or calculator to compute in a problem situation. [SP, RL] ■ Use counting strategies to combine whole numbers with sums under 12. [SP, RL]	**Understand and apply strategies and appropriate tools for adding and subtracting with whole numbers.** ■ Use mental math strategies to compute (e.g., composing and decomposing numbers, finding combinations that are easy to add or subtract) through 100. [RL] ■ Use calculator, manipulatives, or paper and pencil to solve addition or subtraction problems. ■ Explain methods to mentally group numbers efficiently (e.g., when adding 52 and 59, add the 50's together to get 100, then add 11 more). [CU]	**Understand and apply strategies and tools as appropriate to tasks involving addition and subtraction on whole numbers.** ■ Use appropriate strategies and tools from among mental computation, estimation, calculators, and paper and pencil to compute in a problem situation. [SP, RL] ■ Defend situations in which estimation is sufficient (e.g., grocery shopping or party supplies). [CU] ■ Use mental arithmetic, pencil and paper, or calculator as appropriate to the task involving addition and subtraction of whole numbers.	**Understand and apply strategies and tools as appropriate to tasks involving multiplication and division on whole numbers.** ■ Select and justify appropriate strategies and tools from among mental computation, estimation, calculators, and paper and pencil to compute in a problem situation. [SP, RL] ■ Use estimation strategies appropriately when the exact answer is not necessary. [SP, RL] ■ Identify and justify situations when estimation is not appropriate. [SP, RL, CU, MC] ■ Use mathematical tools as appropriate to the task involving multiplication and division of whole numbers.
Estimation					
1.1.8		**Understand and apply estimation strategies to determine the reasonableness of answers.** ■ Use a known quantity (e.g., chunking) to make reasonable estimates. [RL] ■ Use numbers that are easy to add or subtract to make a reasonable estimate of a sum (e.g., 9 + 8 should be about 20, since 9 is about 10, 8 is about 10, and 10 + 10 is 20). [RL]	**Understand and apply estimation strategies to predict computation results and to determine the reasonableness of answers.** ■ Use estimation strategies (e.g., front-end estimation, clustering) to predict computation results and to determine the reasonableness of answers. [RL] ■ Justify reasonableness of an estimate in addition and subtraction. [CU] ■ Decide whether a given estimate for a sum or difference is reasonable. [RL]	**Understand and apply estimation strategies to determine the reasonableness of answers in situations involving addition and subtraction on whole numbers.** W ■ Identify when an approximation is appropriate. ■ Use estimation to determine the reasonableness of answers in situations. [RL] ■ Describe and justify reasonableness of an estimate in computation. [RL, CU] ■ Use a variety of estimation strategies (e.g., multiples of 10 and 100, rounding, front-end estimation, compatible numbers, clustering). ■ Describe and justify whether an approximation is or is not appropriate. [RL, CU]	**Understand and apply estimation strategies to determine the reasonableness of answers in situations involving multiplication and division on whole numbers.** W ■ Identify when an approximation is appropriate. ■ Use a variety of strategies to approximate sums, differences, products, and quotients. [RL] ■ Use estimation to determine the reasonableness of answers in situations. [RL] ■ Make and explain an appropriate adjustment when an estimate and a solution don't agree. [RL, CU]

SP: Solves Problems RL: Reasons Logically CU: Communicates Understanding MC: Makes Connections

GLE	5	6	7	8	9/10
Computation					
1.1.7	**Understand and apply strategies and tools as appropriate to tasks involving addition and subtraction of non-negative, like-denominator fractions, or decimals.** ■ Select and justify strategies and appropriate tools from among mental computation, estimation, calculators, manipulatives, and paper and pencil to compute a problem situation. [SP, RL] ■ Use mental arithmetic to add and subtract non-negative decimals and like-denominator fractions.	**Understand and apply strategies and tools to complete tasks involving addition and subtraction on non-negative rational numbers.** ■ Select and justify the selection of appropriate strategies and tools (e.g., mental computation, estimation, calculators, and paper and pencil) to compute in a problem situation. [SP, CU] ■ Describe strategies for mentally solving problems involving fractions and decimals. [CU] ■ Use calculators to add and subtract with decimal numbers with precision to the thousandths place and beyond.	**Understand and apply strategies and tools to complete tasks involving addition and subtraction on integers and the four basic operations on non-negative rational numbers.** ■ Select and justify the selection of appropriate strategies and tools (e.g., mental computation, estimation, calculators, and paper and pencil) to compute in a problem situation. [SP, RL] ■ Convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator. [MC] ■ Use calculators to add and subtract with integers of two or more digits. ■ Use calculators to compute with decimal numbers with precision from the thousandths place and beyond.	**Understand and apply strategies and tools to complete tasks involving computation on rational numbers.** ■ Select and justify appropriate strategies and tools (e.g., mental computation, estimation, calculators, and paper and pencil) to compute in a problem situation. [SP, RL] ■ Describe strategies for mentally solving problems involving integers and exponents. [CU] ■ Use calculators to compute with whole number powers beyond the cubed numbers. ■ Use calculators to compute square roots of perfect squares greater than 100.	
Estimation					
1.1.8	**Understand and apply estimation strategies to determine the reasonableness of answers in situations involving addition and subtraction on non-negative decimals and like-denominator fractions.** W ■ Identify when an approximation is appropriate. ■ Use estimation strategies prior to computation of addition and subtraction of decimals and like-denominator fractions to predict answers. [RL] ■ Use estimation to determine the reasonableness of answers in situations. ■ Determine reasonableness of estimated answers for a given situation. [RL] ■ Demonstrate or explain various strategies used during estimation. [CU]	**Apply estimation strategies to predict or determine the reasonableness of answers in situations involving addition and subtraction on non-negative rational numbers.** W ■ Identify when an approximation is appropriate. [MC] ■ Apply estimation strategies prior to computation on whole numbers, decimals, and fractions to approximate an answer. [RL] ■ Use estimation to verify the reasonableness of calculated results. [RL] ■ Identify appropriate estimated answers for a given situation. ■ Describe various strategies used during estimation involving fractions and decimals. [CU]	**Apply estimation strategies to predict or determine the reasonableness of answers in situations involving addition and subtraction of integers and the four basic operations on non-negative rational numbers.** W ■ Identify when an approximation is appropriate in situations. [MC] ■ Use estimation strategies prior to operations on non-negative rational numbers to approximate an answer. [RL] ■ Justify why estimation would be used rather than an exact computation. [CU] ■ Describe a situation where estimation is sufficient in real-life contexts. [CU, MC] ■ Use estimation to verify the reasonableness of calculated results. [RL] ■ Evaluate the appropriateness of estimation in a situation and support the evaluation. [RL]	**Apply estimation strategies to predict or determine the reasonableness of answers in situations involving computation on rational numbers in any form including whole number powers and square roots of square numbers.** W ■ Identify when an approximation is appropriate. [MC] ■ Explain situations involving rational numbers where estimates are sufficient and others for which exact value is required. [CU] ■ Justify why an estimate would be used rather than an exact answer in a given situation. [CU] ■ Describe various strategies used during estimation involving integers. [CU] ■ Use estimation to predict or to verify the reasonableness of calculated results. [RL]	**Apply estimation strategies to determine the reasonableness of results in situations involving multi-step computations with rational numbers including whole number powers and square and cube roots.** W ■ Identify when an approximation is appropriate. [MC] ■ Explain situations involving real numbers where estimates are sufficient and others for which exact value is required. [CU] ■ Justify why an estimate would be used rather than an exact answer in a given situation. [CU] ■ Describe various strategies used during estimation involving integers, rational numbers. [CU] ■ Use estimation to predict or to verify the reasonableness of calculated results. [RL]

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.2: Understand and apply concepts and procedures from measurement.

GLE	K	1	2	3	4
Attributes, units, and systems					
1.2.1	**Understand and apply appropriate terminology to compare attributes.** ■ Use comparative vocabulary to describe objects (e.g., longer/shorter, heavier/lighter, nearer/further, thicker/thinner, shorter/taller). [CU] ■ Use terms to describe the duration of events (e.g., long time or short time). [CU] ■ Identify and sort objects based on an attribute (e.g., color, shape, texture). [RL]	**Understand and apply attributes to describe and compare objects.** ■ Order three or more objects according to an attribute (e.g., pencil lengths, students' hand span, and thickness of books). [RL] ■ Read a clock with only the hour hand and use approximate language (e.g., almost 7, a little after 7). [CU] ■ Identify coins (penny, nickel, dime, quarter) and state their value. [CU]	**Understand and apply attributes to measure objects and time.** ■ Identify attributes of an object that are measurable (e.g., time, length, distance around, or weight of objects). ■ Compare lengths or distances where direct comparison is not possible (e.g., use a string, paper strip, arm length, or hand span to compare the height and width of a table). [RL, MC] ■ Read a clock to tell time to the half hour.	**Understand how different attributes (length, perimeter, time, money value, weight/mass, and temperature) are used to describe objects.** W ■ Given an object, name the attributes that can be measured. [CU, MC] ■ Explain how length is used to describe objects. [CU] ■ Explain or show how height and weight are different. [CU] ■ Explain or show how clocks measure the passage of time. [CU] ■ Explain how money is used to describe the value of purchased items. [CU]	**Understand the concept of area.** W ■ Demonstrate and explain how area covers a shape and perimeter encloses a shape. [CU, MC] ■ Describe situations where area is the needed measurable attribute (e.g., buying carpet to cover a floor, painting a wall, building fishnets based on fishing ground, calculating needed area for teepees and lodges, amount of area needed for a pow-wow, describing the amount of floor space in a room). [CU, MC] ■ Compare areas of different shapes and sizes. [RL] ■ Use measurements of area to describe objects. [CU]
1.2.2				**Understand the differences between non-standard and standard units of measurement for length and weight/mass in either U.S. or metric systems.** W ■ Identify when two unit measurements are not necessarily equal (e.g., one pace long can represent different lengths). [CU, MC] ■ Determine whether measurement can or cannot be compared based on whether the units are the same or different. ■ Show how length units are shown on rulers, tape measures, and other linear measuring tools. [MC, CU] ■ Show how weight units are shown on a grocery scale. [MC] ■ Explain why people created standard units for length or weight/mass. [CU]	**Understand the differences between length units and area (square) units in U.S. or metric systems.** W ■ Measure perimeter and area for regular and irregular shapes (e.g., use tiles, inches, or grid paper to find perimeter or area of mats, CDs, or skateboards). [SP, RL, MC] ■ Compare and describe area measurements made using different units (e.g., square inches vs. square centimeters). [SP, RL] ■ Describe how the unit chosen to measure linear dimensions can determine the unit used to measure area (e.g., measuring perimeter in centimeters produces an area in square centimeters). [CU]

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

Component 1.2: Understand and apply concepts and procedures from measurement.

GLE	5	6	7	8	9/10
Attributes, units, and systems					
1.2.1	**Understand the concept of angle measurement.** W ■ Describe and compare angles in a variety of objects. [CU] ■ Identify angles in the environment. [MC] ■ Classify or sort angles as right, acute, or obtuse. [RL, CU] ■ Identify types of angles in polygons (e.g., right, acute, obtuse). [MC] ■ Explain and provide examples of how angles are formed.	**Understand the concepts of volume and extend the concept of area to surface area of rectangular prisms.** W ■ Compare the relative capacity of two containers and explain the differences (e.g., paper cylinders formed horizontally and vertically and filled with popcorn). [RL] ■ Represent the volume for given rectangular prisms using pictures or models. [CU] ■ Compare the surface area of two different rectangular prisms. ■ Describe and provide examples for surface area measurement (e.g., gift wrapping, painting a room, amount of material needed to build a box). [MC] ■ Explain and give examples of how the area and surface area are related (e.g., surface area is the sums of the areas of all the sides of a rectangular prism). [CU, MC] ■ Describe and compare the use of area and volume (e.g., covering and filling). [CU]	**Analyze how a change in a linear dimension affects other linear measurements (perimeter, circumference) and area measurements.** W ■ Describe the relationships among linear dimensions (e.g., radius of a circle, length of a side or base, changes in the diameter affects the amount of deer hide needed to cover a drum face) and area of the figure (e.g., change the radius or length of a side, and check the change in area; describe that change). [CU] ■ Explain changing one, two, or three dimensions in a rectangular prism and how it affects the surface area and volume; give three examples. ■ Solve problems involving the effects of changes in one dimension on area (e.g., given a garden with certain dimensions, make the area of the garden *x* square units by changing only one dimension of the garden). [SP]	**Analyze how a change in a linear dimension affects volume and surface area of rectangular prisms and right cylinders.** W ■ Compare the impact that a change in one dimension has on volume and surface area in right cylinders and rectangular prisms. [SP, RL] ■ Describe the relationships among linear dimensions, volume, and surface area (e.g., changing the length of a side affects the surface area and volume). [CU] ■ Solve problems involving the effects of changes in one dimension on area (e.g., given a box with certain dimensions, make the volume of the box *y* cubic units by changing only one dimension of the box). [SP]	**Analyze how changes in one or two dimensions of an object affect perimeter, area, surface area, and volume.** W ■ Describe and compare the impact that a change in one or more dimensions has on objects (e.g., how doubling one dimension of a cube affects the surface area and volume). [CU, MC] ■ Describe how changes in the dimensions of objects affect perimeter, area, and volume in real-world situations (e.g., how does the change in the diameter of an oil drum affect the area and volume?). [CU, MC] ■ Solve problems by deriving the changes in two dimensions necessary to obtain a desired surface area and/or volume (e.g., given a box with certain dimensions, make the volume of the box y cubic units by changing two dimensions of the box). [SP] ■ Compare a given change in one or two dimensions on the perimeter, area, surface areas, or volumes of two objects. ■ Determine the change in one dimension given a change in perimeter, area, volume, or surface area.
1.2.2	**Understand degrees (30°, 45°, 60°, 90°, and 180°) as units of measurement for angles.** W ■ Describe an angle in relation to a right angle. [RL] ■ Measure angles to the nearest 5 degrees using a protractor, angle ruler, or other appropriate tool. [RL] ■ Measure angles in assorted polygons and determine the total number of degrees in the polygon. [SP, RL] ■ Explain how degrees are used as measures of angles (e.g., a circle can be divided into 360°). ■ Identify, draw, or demonstrate angles that match or approximate 30°, 45°, 60°, 90°, and 180°. [CU]	**Understand the differences between square and cubic units.** W ■ Identify cubic units to measure volume (e.g., linking cubes, cubic centimeter). ■ Identify and read incremental units for capacity (e.g., milliliters, cups, ounces). ■ Use the appropriate units when describing a situation (e.g., five square meters of carpet, five cubic meters of water). [MC] ■ Explain why volume is measured in cubic units. [CU, MC] ■ Explain how the selected unit of length affects the size of cubic units (e.g., centimeter versus inch). [CU]		**Understand and apply derived units of measurement.** W ■ Explain the concept of a rate. [CU] ■ Explain how division of measurements produces a derived unit of measurement (e.g., miles traveled divided by hours traveled yields the derived unit [miles per hour]). [CU] ■ Find a rate of change in a situation (e.g., increase per year in stamp cost) and label the results. [SP, RL, MC] ■ Use unit analysis to find equivalent rates (e.g., miles per hour to feet per second). [MC] ■ Use rate to determine a measured outcome.	

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.2: Understand and apply concepts and procedures from measurement.

GLE	K	1	2	3	4
Attributes, units, and systems					
1.2.3				**Understand how measurement units of length (U.S.) and capacity (U.S.) are organized into systems.** W ■ Describe the various units of measurement for length and capacity and explain how they are organized. ■ Explain the benefits and appropriate uses of standard units of measurement for length and capacity using our customary (U.S.) system. [CU] ■ Demonstrate or explain how inches are organized into feet and feet are organized into yards. [CU] ■ Demonstrate or explain how cups are organized into pints, pints into quarts, and quarts into gallons. [CU]	**Understand how measurement units of time and weight (U.S.) are organized into systems.** W ■ Know and correctly label the basic units of measurement for time and weight measure in the metric and customary system. [CU] ■ Explain the benefits and appropriate uses of standard units of measurement for area using both customary and metric systems. [CU] ■ Demonstrate or explain how seconds are organized into minutes, minutes into hours, hours into days, days into weeks, and weeks into years. [CU] ■ Demonstrate or explain how months are organized into years. [CU] ■ Demonstrate or explain how ounces are organized into pounds. [CU]
Procedures, precision, and estimation					
1.2.4	**Understand and apply procedures to measure with non-standard units.** ■ Use non-standard units to measure (e.g., paper strips, cubes, beans, hand widths). ■ Explain how to use a non-standard unit to measure a given length (e.g., length of a table, width of a desk). [CU]	**Understand and apply procedures to measure with non-standard or standard units.** ■ Select units appropriate to the object being measured (e.g., measure length of classroom with footprints, not beans) and explain why it was selected. [CU] ■ Use a uniform unit to measure an object (e.g., cubes, paper strips, ruler). ■ Measure a variety of objects using appropriate non-standard tools (e.g., arm length, hand width, lengths of rope). ■ Use a variety of records of time (e.g., calendar, seasonal plants, animal migrations, moon phases, tides, shadows). ■ Use physical models of measuring units to fill, cover, match, or make the desired comparison of the attribute with the unit. [SP, RL] ■ Explain the need for appropriate tools for measurement. [CU]	**Understand and apply procedures to measure with non-standard or standard units.** ■ Select the most appropriate unit to measure the time of a given situation (e.g., would you use minutes or hours to measure brushing your teeth, eating dinner, sleeping?). [MC] ■ Select a tool that can measure the given attribute (e.g., analog clock—time, string—length, balance—weight). ■ Demonstrate measurement procedure (e.g., start at a beginning point, place units end-to-end, not overlapping, and straight line). [CU] ■ Justify the use of one tool over another (e.g., the length of a hand is a better measurement tool for this situation than the length of a small cube). [CU, RL] ■ Explain why, when the unit is smaller, it takes more to measure an item than when the unit is larger (e.g., it takes more small paper clips than large paper clips to measure the same length). [CU]	**Understand and apply systematic procedures to measure length, time, weight, money value, and temperature.** W ■ Identify attribute to measure. ■ Select and use appropriate units (e.g., meters, minutes, pounds, dollars, degrees). ■ Select and use tools that match the unit (e.g., ruler, clock, scales, calculator, thermometer). ■ Count or compute and label measures. ■ Explain and use a method for making change with coins. [CU]. ■ Compare measures of two or more like objects. [RL]	**Understand and apply systematic procedures to determine the area of figures composed of rectangles.** W ■ Select and use appropriate units (e.g., square units). ■ Select and use tools that match the unit (e.g., grid paper, squares). ■ Count or compute and label area measures. ■ Explain and use a method for measuring the area of an irregular shape (e.g., describe an irregular shape in terms of the composition of regular figures). [CU] ■ Solve problems involving area measurement. [SP] ■ Analyze a measurement situation and determine whether measurement has been done correctly. [RL]

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

GLE	5	6	7	8	9/10
Attributes, units, and systems					
1.2.3	**Understand how measurement units of capacity, weight, and length are organized in the metric system.** W ■ Explain and give examples of the metric system standard units for capacity, weight, and length. ■ Demonstrate or explain how grams are organized into kilograms. [CU] ■ Demonstrate or explain how millimeters are organized into centimeters and how centimeters are organized into meters. [CU] ■ Demonstrate or explain how milliliters are organized into liters. [CU]		**Understand how the unit of measure affects the precision of measurement.** W ■ Select the appropriate measurement tool to match the precision needed (e.g., if needing measurement to the nearest 1/16 inch, select a ruler that has 1/32 increments). ■ Explain how the unit selected for a situation can affect the precision of the measurement (e.g., when you have a ruler that has only 1/10 increments, you cannot measure something to the nearest hundredth with confidence of precision). ■ Explain how measurement systems allow for different levels of precision (e.g., millimeters give more precise measurement than centimeters). [CU]	**Understand why different situations require different levels of precision.** W ■ Explain the relationships among units within both the customary and metric system (e.g., kilograms to grams, feet to inches). ■ Justify the use of a unit of measure (e.g., measuring to order fencing requires a different precision than if one is selling land and needs to be precise about borders). [CU, MC] ■ Compare situations for the level of precision needed. [RL] ■ Explain and give examples of situations that require more and less precision. [CU]	**Understand how to convert units of measure within systems (U.S. or metric).** W ■ Understand how to convert units of measure within U.S. or within metric systems to achieve an appropriate level of precision. ■ Convert within a system to a unit size appropriate to a given situation. ■ Convert to a larger unit within a system while maintaining the same level of precision (e.g., represent 532 centimeters as 5.32 meters). ■ Convert to a smaller unit within a system to increase the precision of a derived unit of measurement.
Procedures, precision, and estimation					
1.2.4	**Understand and apply systematic procedures to determine the areas of rectangles and right triangles.** W ■ Select and use appropriate units for measuring area (e.g., square units) or dimensions. ■ Select and use tools that match the unit (e.g., grid paper, squares, ruler). ■ Explain a method for measuring the area of a rectangle or right triangle (e.g., use the formula for the area of a rectangle or triangle, select grid paper). [CU] ■ Use measurements of area to describe and compare rectangles or triangles. ■ Solve problems involving measurement of area in rectangle and triangle (e.g., create a design using triangles and rectangles and determine how much paint is needed to cover the area of each of the shapes). [SP] ■ Analyze a measurement situation and determine whether measurement has been done correctly. [RL]	**Understand and apply systematic procedures to measure volume and capacity for solid shapes.** W ■ Identify the attribute to be measured in the situation (e.g., volume or capacity). ■ Choose the appropriate standard unit for measuring volume or capacity (e.g., cubic inches vs. cubic feet, cups vs. gallons). ■ Select and use tools that match the unit. ■ Count or compute to obtain the volume or capacity and label the measurement. ■ Use volume and capacity to describe and compare figures (e.g., fill containers with cubes to find which has a greater volume). [RL, CU] ■ Measure the capacity of containers using appropriate tools and label (e.g., graduated cylinders, measuring cups, tablespoons). [CU] ■ Evaluate whether measurement has been done correctly. [RL]			

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.2: Understand and apply concepts and procedures from measurement.

GLE	K	1	2	3	4
Procedures, precision, and estimation					
1.2.5					
1.2.6			**Understand how to estimate in measurement situations.** ■ Estimate length and weight using non-standard units. [RL] ■ Use important benchmarks (referents) (e.g., 5 or 10) to make initial and revised estimates. ■ Explain how a benchmark (referent) helps to make a reasonable estimate. [CU]	**Understand and apply strategies to obtain reasonable estimates of length, time, weight, and temperature measurements.** W ■ Identify situations in which estimated measurements are sufficient; estimate length, time, money, weight or temperature. ■ Estimate a measurement using standard or non-standard units (e.g., fingers, arms, paper clips, inches, minutes, or foot lengths). ■ Create and use referents to standard units (e.g., width of pinkie finger is similar to a centimeter). [RL, MC] ■ Use estimation to decide whether standard or non-standard units of measurement have been used in a situation. [RL] ■ Determine when estimation is useful.	**Understand and apply strategies to obtain reasonable estimates of area measurements for irregular figures.** W ■ Identify situations in which estimate measurements are sufficient. ■ Apply a process that can be used to find a reasonable estimate of the area measurement of an irregular shape (e.g., use tiles or pieces of paper to measure leaves, ponds). [SP, RL, CU] ■ Compare areas of irregular shapes with different perimeters (e.g., leaves, ponds). [RL, MC] ■ Explain whether estimation or precision is needed in a given situation. [CU] ■ Determine whether a given measurement is exact or an estimate.

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

Component 1.2: Understand and apply concepts and procedures from measurement.

GLE	5	6	7	8	9/10
Procedures, precision, and estimation					
1.2.5	**Understand and apply formulas to measure area and perimeter of rectangles and right triangles.** W ■ Explain how to find the perimeter or area of any rectangle using a rule. [CU] ■ Explain and use formulas to find the perimeter or area of a rectangle. [CU] ■ Explain and use a formula to find the area of a right triangle. [CU] ■ Find and compare all possible rectangles or right triangles with whole number dimensions with a given perimeter or area (e.g., a rectangle with an area of 24 square feet could be 1'x24', 2'x12',3'x8', or 4'x6'). [RL, CU] ■ Explain why formulas are used to find area and/or perimeter. [CU]		**Apply formulas to find measurements of circles, triangles, and rectangular prisms.** W ■ Apply formulas to determine missing measurements for circles, rectangular prisms, and triangles. ■ Explain how to use a formula for finding the area and circumference of a circle (e.g, calculate the area needed to cover a drum face). [CU] ■ Find and compare the volumes of rectangular prisms that have a given volume (e.g., if two rectangular prisms have the same volume and one has twice the height of the other, determine how the areas of their bases compare). [RL] ■ Justify the standard formula for finding the area of a right triangle (e.g., 1/2 of a rectangle). [CU] ■ Use given dimensions to determine surface area and volume.	**Understand and apply formulas including the Pythagorean Theorem to right prisms, right cylinders, and triangles.** W ■ Explain how to use a formula for finding the surface area and volume of a solid. [CU] ■ Find missing sides or area of right triangles (e.g., use the Pythagorean Theorem to find any of the missing values). ■ Calculate measures of objects for which no direct information is given (e.g., apply ratio, proportion, and scale to determine the area, surface area, and/or volume of a similar figure or solid). [SP, MC] ■ Compare surface areas of shapes with given volumes (e.g., compare cost of material to make various right cylinder and right prism containers with a given volume). [RL, MC]	**Apply formulas to calculate measurements of right prisms or right circular cylinders.** W ■ Explain how to use a formula for finding the volume of a prism or cylinder. [CU, MC] ■ Use a formula to find the volume of a prism or cylinder. [RL, MC] ■ Use a formula to derive a dimension of a right prism or right cylinder given other measures. ■ Use formulas to describe and compare the surface areas and volumes of two or more right prisms and/or right cylinders. [RL] ■ Use formulas to obtain measurements needed to describe a right cylinder or right prism.
1.2.6	**Understand and apply strategies to obtain reasonable estimates of angles and area measurements for rectangles and triangles.** W ■ Identify situations in which estimated measurements are sufficient. ■ Estimate measures of angles and areas in rectangles and triangles. ■ Estimate a measurement using standard or non-standard units (e.g., tiles, square feet, note cards). ■ Use estimation to justify reasonableness of a measurement (e.g., estimate the area of the classroom by using carpet squares). [RL] ■ Determine whether an angle is closest to 30°, 45°, 60°, 90°, or 180°. ■ Explain or identify an appropriate process for estimating area or angle measurement. [CU]	**Understand and apply strategies to obtain reasonable estimates of volume or capacity.** W ■ Identify situations in which estimated measures are sufficient. ■ Estimate volume or capacity. ■ Use estimation to justify reasonableness of a volume of a rectangular prism. [RL] ■ Estimate a measurement of volume or capacity using standard or non-standard units (e.g., estimate the capacity of a bowl in cups and handfuls). [SP] ■ Use or describe a process to find a reasonable estimate of volume or capacity (e.g., fill a container with rice or popcorn). [CU]	**Understand and apply strategies to obtain reasonable estimates of circle measurements, right triangles, and surface area for rectangular prisms.** W ■ Identify situations in which estimated measures are sufficient. [MC] ■ Estimate circle and triangle measurements. ■ Use common approximations of pi (3.14; 22/7) to calculate the approximate circumference and the area of circles. ■ Use or describe a process to find a reasonable estimate of circle measurements (e.g., wrap a string around it). [RL] ■ Explain why estimation or precise measurement is appropriate in a given situation. [CU]	**Apply strategies to obtain reasonable estimates of volume and surface area measurements for right cylinders, right prisms, and of the lengths of sides of right triangles.** W ■ Estimate volume and surface area for right cylinders and right prisms. ■ Estimate the length of the remaining side of a right triangle given the lengths of two sides. ■ Approximate distance or height in a problem situation using similar triangles or Pythagorean relationships (e.g., height of a flagpole using proportional reasoning, distance across a lake using Pythagorean relationship). [SP] ■ Use or describe a process for finding area of a right triangle.	**Understand and apply strategies to obtain reasonable measurements at an appropriate level of precision.** W ■ Identify situations in which approximate measurements are sufficient. ■ Estimate a reasonable measurement at an appropriate level of precision. [MC] ■ Estimate quantities using derived units of measure (e.g., distance or time using miles per hour, cost using unit cost). [MC] ■ Estimate derived units of measure (e.g., miles/hour, people/year, grams/cubic centimeter). [MC] ■ Apply a process that can be used to find a reasonable estimate for the volume of prisms, pyramids, cylinders, and cones. ■ Estimate volume and surface area for right cylinders and right prisms.

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.3: Understand and apply concepts and procedures from geometric sense.

GLE	K	1	2	3	4
Properties and relationships					
1.3.1				**Understand the concept of congruence.** W ■ Identify, describe, and compare congruent two-dimensional geometric figures. [RL, CU] ■ Given a variety of figures, determine which figures are congruent. ■ Draw a shape that is congruent to a given two-dimensional shape. [CU] ■ Explain congruence and use an example to demonstrate it. [CU]	**Understand concepts of parallel and perpendicular lines and line symmetry in two-dimensional shapes and figures.** W ■ Identify symmetrical two-dimensional figures and shapes (e.g., quilt blocks, textiles). [CU] ■ Complete a picture or design from a variety of cultures that incorporate a line of symmetry (e.g., basket design, beadwork, quilts, pyramids, nature). ■ Identify and draw a line of symmetry (e.g., folding or using a mirror). [CU] ■ Identify parallel and perpendicular lines in two-dimensional figures and shapes and in the environment. [MC] ■ Describe characteristics of two-dimensional geometric figures using appropriate vocabulary of parallel, perpendicular, symmetric (e.g., the U.S. flag, a stop sign, a yield sign, a race track, a football field). [CU, MC] ■ Explain parallel and perpendicular and give examples to demonstrate them. [CU]

GLE	5	6	7	8	9/10
Properties and relationships					
1.3.1	**Understand properties of angles and polygons.** W ■ Explain the difference between a regular and irregular polygon. [CU] ■ Identify, sort, classify, or explain the properties of angles, polygons, or circles based on attributes (e.g., triangles [right, equilateral, isosceles, or scalene], angles [acute, right, obtuse, or straight], or quadrilaterals [squares, rectangles, parallelograms, or trapezoids]). [RL, CU] ■ Construct a geometric shape using geometric properties. [MC]	**Understand the characteristics of circles and rectangular prisms.** W ■ Name and sort circles or rectangular prisms according to their attributes (faces, edges, radii, base, parallel faces). [RL] ■ Draw a figure with given characteristics (e.g., the set of points equidistant from a given point). [CU] ■ Identify lines of symmetry in rectangular prisms. ■ Explain lines of symmetry for circles. [CU] ■ Describe the relationship between the diameter and the radius of a circle. [CU]	**Understand the concept of similarity.** W ■ Identify corresponding sides and angles of two similar figures. ■ Determine and justify if two figures are similar using the definition of similarity. [CU, RL] ■ Differentiate between similar and congruent figures, either geometric figures or real-world objects, and justify the conclusion. [RL, MC] ■ Explain how a scale drawing is an example of similarity. [CU]	**Apply understanding of characteristics and relationships among one-dimensional, two-dimensional, and three-dimensional figures to solve problems.** W ■ Identify and label rays, lines, end points, line segments, vertices, and angles. [CU] ■ Match or draw three-dimensional objects from different perspectives using the same properties and relationships (e.g., match to the correct net, draw the top view). [RL] ■ Draw and label with names and symbols, nets of prisms, and cylinders. [RL, CU] ■ Describe everyday objects in terms of their geometric characteristics. [CU] ■ Identify the two-dimensional components of three-dimensional figures.	**Understand the relationship among characteristics of one-dimensional, two-dimensional, and three-dimensional figures.** W ■ Identify and label one- and two-dimensional characteristics (rays, lines, end points, line segments, vertices, and angles) in three-dimensional figures. [CU] ■ Match or draw three-dimensional objects from different perspectives using the same properties and relationships (e.g., match to the correct net, draw the top view). [RL] ■ Draw and label with names and symbols nets of right prisms and right cylinders. [RL, CU] ■ Describe everyday objects in terms of their geometric characteristics. [CU] ■ Describe or classify various shapes based on their characteristics. ■ Make and test conjectures about two-dimensional and three-dimensional shapes and their individual attributes and relationships using physical, symbolic, and technological models (e.g., diagonal of a rectangle or prism is the longest interior segment; what figures make up cross-sections of a given three-dimensional shape?). [SP, RL, CU, MC]

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.3: Understand and apply concepts and procedures from geometric sense.

GLE	K	1	2	3	4
Properties and relationships					
1.3.2	**Know the characteristics of familiar objects.** ■ Describe familiar objects based on characteristics (e.g., big, small, like a box). [CU, MC] ■ Sort objects in their environment by characteristics (e.g., cans, balls, boxes, red, blue). [MC] ■ Describe objects using comparative language (e.g., bigger, taller, shorter, smaller). [CU]	**Understand how to compare figures based on their characteristics.** ■ Describe two-dimensional figures based on their characteristics (e.g., number of sides, number of equal sides). [CU] ■ Identify, compare, and sort two-dimensional figures in their surroundings (e.g., by lengths of sides, general shape). [RL, MC] ■ Describe figures using accurate terminology (e.g., square, rectangle, triangle).	**Understand characteristics of two-dimensional geometric figures.** ■ Sort and describe characteristics of two-dimensional geometric figures (e.g., various polygons). [RL, CU] ■ Draw a two-dimensional shape that matches a set of characteristics (e.g., draw a four-sided shape that has all sides the same length).	**Understand and apply attributes and properties to two-dimensional shapes and figures.** W ■ Use attributes and properties to identify, name, draw, compare, and/or sort two-dimensional shapes and figures. [RL, CU] ■ Draw and label two-dimensional figures given particular attributes (e.g., triangle, rectangle with all sides the same length). [CU] ■ Identify, name, and describe the attributes and properties of polygons. [CU] ■ Given two polygons, explain how they are alike and different in terms of their attributes and properties (e.g., using a Venn diagram). [CU] ■ Give directions so that someone else can duplicate a design involving polygons (e.g., a friend who can't see the design). [CU]	**Apply understanding of congruence to two-dimensional shapes and figures.** W ■ Identify, describe, and compare attributes of congruent figures in multiple orientations. [CU, SP, RL] ■ Build and draw congruent figures. [CU] ■ Identify, name, compare, and sort congruent two-dimensional figures and shapes in multiple orientations. [RL] ■ Solve problems involving congruence (e.g., create a design made out of congruent shapes). [SP]

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

GLE	5	6	7	8	9/10
Properties and relationships					
1.3.2	**Apply understanding of the properties of parallel and perpendicular and line symmetry to two-dimensional shapes and figures.** W ■ Identify, name, compare, and sort parallel and perpendicular lines in two-dimensional figures. [SP, RL, CU] ■ Draw and label a design that includes a given set of attributes (e.g., create a design that has only two lines of symmetry; parallel and perpendicular lines). [SP, CU] ■ Sort figures based on characteristics of parallel lines, perpendicular lines, and/or lines of symmetry. ■ Draw figures or shapes that have particular characteristics (e.g., create a figure that has two parallel lines and one line of symmetry). ■ Identify parallel and perpendicular lines and/or lines of symmetry in the environment. ■ Construct a geometric shape using given geometric properties. [CU] ■ Use technology to draw figures with given characteristics. [MC]	**Apply understanding of angles and polygons.** W ■ Identify geometric figures and concepts in nature and art (e.g., triangle in architecture, rhombus in beadwork, culturally relevant textiles, quilts). [MC] ■ Combine polygons to create given two-dimensional figures and represent them on grid paper (e.g., use all pieces of tangrams to create a square). [SP, RL, CU] ■ Create a three-dimensional shape given its net or draw the net of a given three-dimensional shape. [RL] ■ Find the missing measure of an angle using the properties of parallel lines, perpendicular lines, vertical and corresponding angles. ■ Find the missing angle given all but one of the angles of a polygon. [RL]	**Apply understanding of the characteristics of rectangular prisms and circles.** W ■ Identify, describe, compare, and sort figures. ■ Draw rectangular prisms and circles with specified properties (e.g., circumference of an 18-centimeter quadrilateral having equal sides but no right angles; a triangle with no equal sides). [CU] ■ Use the properties of rectangular prisms and circles to solve problems (e.g., determine which of two rectangular prism-shaped boxes will hold the most cans of food at the food drive and explain how the geometric characteristics affect capacity). [SP, RL, CU, MC] ■ Compare two rectangular prisms based on their characteristics (e.g., compare the geometric characteristics of two rectangular prisms with different dimensions and the same volume). [RL]	**Apply understanding of similarity to two-dimensional figures.** W ■ Use properties of similarity to draw, describe, and compare two-dimensional figures. ■ Find the length of a missing side or the measure of a missing angle of one of the figures, given two similar figures. [SP, RL] ■ Create symmetrical, congruent, or similar figures using a variety of tools (e.g., ruler, pattern blocks, geoboards). [RL, CU] ■ Draw a similar shape to a given shape. [RL, CU, MC] ■ Use properties of circles, cylinders, and figures with rotational symmetry to compare figures. [RL, CU] ■ Create a scale drawing and label the scale and the dimensions. (SP, CU, MC).	**Apply understanding of geometric properties and relationships.** W ■ Use geometric properties and relationships to describe, compare, and draw two-dimensional and three-dimensional shapes and figures. ■ Construct geometric figures using a variety of tools and technologies (e.g., angle bisectors, perpendicular bisectors, triangles given specific characteristics). [MC] ■ Draw a plane shape and justify the answer given a set of characteristics. [RL, CU] ■ Use the properties of two-dimensional and three-dimensional shapes to solve mathematical problems (e.g., find the width of a river based on similar triangles; given a set of parallel lines, a transversal, and an angle, find the other angles). [SP, RL, CU, MC] ■ Compare two-dimensional and three-dimensional shapes according to characteristics including faces, edges, and vertices, using actual and virtual modeling. [RL, CU] ■ Use technology to generate two- and three-dimensional models of geometric figures with given geometric characteristics (e.g., generate a two-dimensional animation using pentagons with fixed coordinates for one edge). [RL, SP] ■ Create a three-dimensional scale drawing with particular geometric characteristics. [SP, CU, MC]

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.3: Understand and apply concepts and procedures from geometric sense.

GLE	K	1	2	3	4
Locations and transformations					
1.3.3	**Understand the relative position of objects in the environment.** ■ Describe the location of an object relative to another (e.g., in, out, over, under, behind, above, below, next to, etc.). [CU] ■ Identify where a three-dimensional object is located relative to another given object (e.g., where the eraser is relative to the desk).	**Understand the locations of numbers on a positive number line.** ■ Indicate whether a number is above or below a benchmark number (e.g., greater than or less than 100). ■ Describe the location of a given number between 1 and 100 on a number line. [CU] ■ Identify a point up to 100 on a positive number line.	**Understand the locations of numbers on a positive number line.** ■ Indicate whether a number is above or below a benchmark number (e.g., greater than or less than 1000). ■ Describe the location of a given number between 1 and 1000 on a number line. [CU] ■ Identify a point up to 1000 on a positive number line.	**Understand relative locations including intervals of numbers on a positive number line.** W ■ Given directions for movement on a positive number line, identify the point of final destination using real-world examples (e.g., travel back and forth on a street, temperature variation at different times of the day, dance steps from diverse cultures). [SP, RL, MC] ■ Identify the interval on a given number line (e.g., describe the scale on a graph). [CU] ■ Describe the relative locations of points on a number line with positive coordinates. [CU] ■ Use unit values to describe the location of objects on a number line. ■ Draw points or objects on a number line based on unit values given.	**Apply understanding of the location of points on a coordinate grid in the first quadrant.** W ■ Describe the location in the first quadrant on a coordinate grid in terms of horizontal and vertical position (e.g., to the right and up, longitude and latitude). [CU, MC] ■ Plot a given set of ordered pairs in the first quadrant of a coordinate grid. [CU] ■ Give directions from one location to another using ordered pairs in the first quadrant of a coordinate grid (e.g., given a state map, specify location of landmarks). [CU, MC]
1.3.4					**Understand and apply single transformations using a translation (slide) or reflection (flip).** W ■ Simulate translations and reflections using objects (e.g., pattern blocks, geo blocks). [MC] ■ Record results of a translation or a reflection (e.g., given a polygon on a grid, translate or reflect it and list the new ordered pairs of the vertices). [CU] ■ Identify and draw a single translation (slide) or a single reflection (flip). [CU] ■ Create designs using translations and/or reflections. [SP]

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

GLE	5	6	7	8	9/10
Locations and transformations					
1.3.3	**Apply understanding of the location of non-negative rational numbers on a positive number line.** W ■ Use a number line to order fractions or decimals from least to greatest (e.g., not limited to a number line marked from 0 to 1). [SP, RL] ■ Explain what the relative position of numbers on a positive number line means (e.g., to the right means greater than). [CU] ■ Identify the appropriate values of points on an incomplete number line involving fractional or decimal increments (e.g., using a ruler, reading a fuel gauge). [CU]	**Understand the relative location of integers on a number line.** W ■ Show the order of a given set of integers on a number line. [CU] ■ Identify the point of final destination given directions for movement on a number line including positive and negative numbers (vertical or horizontal) (e.g., temperature variation at different times of the day, bank accounts, gain and loss of weight). [MC] ■ Determine the distance between any two integers on a number line. [RL] ■ Describe relative location of points and objects on a number line with both positive and negative numbers. [CU] ■ Identify objects on a number line based on given numeric locations.	**Understand the location of points on a coordinate grid in any of the four quadrants.** W ■ Identify the coordinates of the fourth point to make a rectangle given three points. [RL] ■ Plot and label ordered pairs in any of the four quadrants. [CU] ■ Name the coordinates of a given point in any of the four quadrants. ■ Identify objects or the location of objects on a coordinate grid using coordinates or labels. ■ Use technology to locate objects on a two-dimensional grid. ■ Use ordered pairs to describe the location of objects on a grid.	**Understand and apply procedures to find distance between points in two-dimensional representations.** W ■ Locate a missing vertex given the coordinates of the vertices of a regular polygon. [RL] ■ Apply the Pythagorean Theorem to find the length of a side of a right triangle or distance between two points. ■ Explain a method for finding the missing side of a triangle in a real-world setting (e.g., the height of a totem pole or building). [CU] ■ Describe the relationship of any two or more points on a coordinate grid. [CU] ■ Find the distance between two points on a coordinate grid including lines that are non-parallel with either axis (oblique). [RL, MC]	**Apply understanding of geometric properties and location of points.** W ■ Use coordinates to describe or identify the location of objects on coordinate grids. ■ Describe geometric characteristics of two-dimensional objects using coordinates on a grid. [MC] ■ Describe the location of points that satisfy given conditions (e.g., the set of points equidistant from a given point; a point equidistant from a given set of points). [CU] ■ Represent situations on a coordinate grid or describe the location of points that satisfy given conditions (e.g., locate a gas station to be equidistant from given cities; locate a staking point to maximize the grazing area of a tethered goat). [MC, SP, RL] ■ Use tools and technology to draw objects on a coordinate grid based on given conditions. [CU] ■ Identify, interpret, and use the meaning of slope of a line as a rate of change using physical, symbolic, and technological models. [SP, RL, MC]
1.3.4	**Apply understanding of translations (slides) or reflections (flips) to congruent figures.** W ■ Identify a specific transformation as a translation (slide) or reflection (flip). [CU] ■ Given a shape on a grid, perform and draw at least one transformation (i.e., translation or reflection). [SP, RL] ■ Draw congruent figures and shapes in multiple orientations using a transformation. [SP, RL] ■ Explain a series of transformations in art, architecture, or nature. [CU, MC] ■ Record results of a translation or reflection (e.g., plot a set of ordered pairs on a grid that are vertices of a polygon, translate or reflect it, and list the new ordered pairs). [CU, MC] ■ Create designs using translations and/or reflections. [SP]	**Apply understanding of rotations (turns) to two-dimensional figures.** W ■ Apply rotations (turns) of 90^0 or 180^0 to a simple two-dimensional figure. ■ Create a design using (90^0, 180^0, 270^0, 360^0) rotations (turns) of a shape. [SP, MC] ■ Show how a shape has been rotated by 90^0 or 180^0. [CU] ■ Describe a rotation so that another person could draw it. [CU] ■ Identify the coordinates of objects that have been rotated 90^0, 180^0, or 270^0 on a coordinate grid. ■ Determine whether an object has been translated or rotated on a coordinate grid.	**Understand and apply combinations of translations (slides) and reflections (flips) to two-dimensional figures.** W ■ Identify and explain whether a shape has been translated (slid) or reflected (flipped) with or without a grid. [RL, CU] ■ Use transformations to create congruent figures and shapes in multiple orientations. ■ Find the coordinate pairs for a translation or a reflection across an axis given a shape on a coordinate grid. [RL] ■ Match a shape with its image following one or two transformations (sliding or flipping). [RL] ■ Use combinations of translations and reflections to draw congruent figures. [RL] ■ Use ordered pairs to describe the location of an object on a coordinate grid after a translation and reflection. [CU]	**Understand and apply transformations to figures.** W ■ Identify and explain how a shape has been translated, reflected, or rotated with or without a grid (e.g., location of the North Star, rotate the Big Dipper). [CU] ■ Use transformations (rotations, reflections, and translations) to draw or locate congruent two-dimensional figures. [RL, CU] ■ Find the image of a given shape after a combination of transformations. [RL] ■ Tessellate a plane by using transformations. [RL, MC] ■ Create a design using a combination of two or more transformations with one or two two-dimensional figures. [SP, RL]	**Apply understanding of multiple transformations to figures.** W ■ Apply multiple transformations to create congruent and similar figures in any or all of the four quadrants. ■ Use multiple transformations (combinations of translations, reflections, or rotations) to draw an image. [RL] ■ Use dilation (expansion or contraction) of a given shape to form a similar shape. [RL, CU] ■ Determine the final coordinates of a point after a series of transformations. [RL, CU] ■ Examine figures to determine rotational symmetry about the center of the shape. [RL, MC] ■ Define a set of transformations that would map one onto the other given two similar shapes. [SP, RL] ■ Create a design with or without technology using a combination of two or more transformations with one or two two-dimensional figures. [SP, RL] ■ Use technology to create two- and three-dimensional animations using combinations of transformations. [MC, SP, RL]

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.4: Understand and apply concepts and procedures from probability and statistics.

GLE	K	1	2	3	4
Probability					
1.4.1					**Understand when events are certain or impossible and more likely, less likely, or equally likely.** W ■ Identify the likelihood of events and use the vocabulary of probability (e.g., weather, if homework will be assigned, simple games). [CU, MC] ■ Place events in order of likelihood of occurrence (e.g., use a number line marked from 0 to 1). [SP, RL, MC] ■ Distinguish between events that are certain or uncertain. [RL] ■ Place events in order based on their likelihood of occurrence. [RL] ■ Identify or describe possible and impossible events. ■ Determine what events are more likely, less likely, or equally likely to happen given an area model (e.g., a spinner with different-sized sections).
1.4.2					

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

GLE	5	6	7	8	9/10
Probability					
1.4.1	**Understand the likelihood (chance) of events occurring.** W ■ Predict and test how likely it is that a certain outcome will occur (e.g., regions of a spinner, flip of a coin, toss of dice). [SP, RL] ■ Represent the probability of a single event on a scale of 0 to 1. [MC] ■ Given a fair game, create an advantage for one of the players (e.g., if the game is selecting marbles, include more marbles of one color than the other). [SP, RL] ■ Explain the likelihood of a single event. [CU] ■ Determine whether a game for two people is fair. [RL] ■ Create a game that would make it more or less likely for an event to happen. [SP]	**Understand probability as a ratio between and including 0 and 1.** W ■ Determine whether a real-life event has zero probability, 50% probability, or 100% probability of occurring. [MC] ■ Express probabilities as fractions or decimals between 0 and 1 and percents between 0 and 100. [CU] ■ Translate between representations of probability (e.g., translate a probability of 6 out of 16 to 3/8 or 37.5%). [MC]	**Understand the concepts of complementary, independent, and mutually exclusive events.** W ■ Determine and explain when events are mutually exclusive (e.g., your grade on a test is an A, B, or C). [CU, MC] ■ Determine and explain when events are complementary (e.g., a person awake or asleep, you pass or fail a test, coin throw — heads or tails). [CU, MC] ■ Identify or explain when events are complementary, mutually exclusive, or neither (e.g., spinning a 4 or a 5 but with the possibility of spinning 1, 2, 3, or 6) and explain. [CU]	**Understand the concept of compound events.** W ■ Determine and explain when events are compound. [CU] ■ Explain the difference between compound events involving 'and' and 'or' (e.g., rolling a six and rolling an odd number vs. rolling a six or rolling an odd number). [CU]	**Understand the concept of conditional probability.** W ■ Compare the probabilities of dependent and independent events. [CU, MC] ■ Determine and justify whether the outcome of a first event affects the probability of a later event (e.g., drawing cards from a deck with or without replacement). [CU] ■ Explain the difference between dependent and independent events. [CU] ■ Explain and give examples of compound events. [CU]
1.4.2	**Understand and apply the Fundamental Counting Principle to situations.** W ■ Calculate the number of different combinations of different objects (e.g., three shirts and two pants could be combined in 3 x 2 = 6 ways). ■ Describe a situation that might include three different selections combined (e.g., describe a situation that could be calculated by 10 x 10 x 26 — two digits and a letter of the alphabet). [CU]	**Understand various ways to determine outcomes of events or situations.** W ■ Determine and use the probabilities of the outcome of a single event. ■ Represent or describe all possible outcomes of experiments (e.g., an organized list, a table, a tree diagram, or a sample space). [RL, CU] ■ Calculate probability for an event (e.g., pulling colored or numbered balls from a bag, drawing a card, rolling a six on a number cube, spinning a spinner, etc.). ■ Determine all possible outcomes (sample space) of an experiment or event (e.g., all different choices a person has to wear one top and one skirt from three different tops and two different skirts). [CU]	**Understand and apply the procedures for determining the probabilities of multiple trials.** W ■ Calculate the probabilities of independent or mutually exclusive outcomes or events. ■ Calculate the probability of an event given the probability of its complement. ■ Create a game that has an equal probability for all players to win. [SP, MC] ■ Revise a game with unequal probabilities for all players and make it a fair game. [SP, MC] ■ Determine, interpret, or express probabilities in the form of a fraction, decimal, or percent. [CU, MC] ■ Predict the probability of outcomes of experiments and test the predictions. [RL] ■ Predict the probability of future events based on empirical data. [RL]	**Understand and apply the procedures for comparing theoretical probability and empirical results for independent or compound events.** W ■ Calculate the probability of two independent events occurring simultaneously using various methods (e.g., organized list, tree diagram, counting procedures, and area model). ■ Explain the relationship between theoretical and empirical probability of compound events. [CU] ■ Predict the probability of outcomes of experiments and compare the predictions to empirical results. [RL] ■ Design or create a situation that would produce a given probability (e.g., how many of each colored marble would it take to have a given probability of selecting one particular color?). [SP, MC] ■ Design a game using compound probabilities with equal chances of winning for all players. [SP, MC]	**Apply understanding of dependent and independent events to calculate probabilities.** W ■ Determine probabilities of dependent and independent events. [SP] ■ Generate the outcomes and probability of multiple independent and dependent events using a model or procedure (e.g., tree diagram, area model, counting procedures). ■ Generate the outcomes and probability of events using a counting procedure (e.g., the number of license plates that can be made with three letters and three numbers; winning the lottery). [MC] ■ Explain the relationship between theoretical probability and empirical frequency of dependent events using simulations with and without technology. [CU] ■ Create a simple game based on independent probabilities wherein all players have an equal probability of winning. [MC, SP] ■ Create a simple game based on compound probabilities. [MC, SP] ■ Determine the sample space for independent or dependent events.

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.4: Understand and apply concepts and procedures from probability and statistics.

GLE	K	1	2	3	4
Statistics					
1.4.3	**Understand how data can be collected and organized.** ■ Use physical objects or pictures to build bar graphs. [CU] ■ Organize objects into groups before counting them. [RL]	**Understand how data can be organized and displayed.** ■ Display results of data collection by making student-invented and conventional displays. [CU] ■ Construct bar graphs with physical materials and record pictorially (e.g., shoes, cats, crops, egg rolls, tacos). [CU] ■ Collect data related to questions and organize the data into useful categories in familiar situations (e.g., how many students like apples? How many students do NOT like apples?).	**Understand the organization of a graph.** ■ Identify title, horizontal and vertical axes, and key. ■ Construct a bar graph that includes a title, key, and single-unit increment. [CU] ■ Name an appropriate title for a display of data. [CU]	**Understand how to use data collection and display methods to obtain desired information.** W ■ Interpret graphs for comparative information (e.g., find the difference in selected data). [RL, CU, MC] ■ Pose questions and gather data relevant to the questions posed. ■ Design a survey; collect and record data in easy-to-use formats (e.g., use tally marks, make a table). [CU] ■ Organize category data into bar graphs with unit scales for ease of interpretation. [RL] ■ Organize data into picture graphs with unit scales for ease of interpretation. [RL] ■ Determine questions needed to gather data about themselves and their classmates.	**Understand and apply data collection methods to obtain the desired information.** W ■ Identify appropriate questions and populations to obtain the desired kind of information. ■ Formulate questions for surveys and collect data. [CU] ■ Decide whether to conduct a survey, use observations, or measure for a given question. [RL] ■ Make a plan to answer a question including how to record and organize data. [RL, CU, MC] ■ Determine which of several questions is most likely to give the desired information. [RL]
1.4.4				**Understand and apply mode to describe a set of data.** W ■ Create and solve a problem situation where mode is meaningful for a set of data. [RL, CU, MC] ■ Explain what the mode represents and how to find it in a given set of data. [CU] ■ Identify the mode for a given set of data	**Understand and apply median and range to describe a set of data.** W ■ Use a variety of strategies to determine median and range from a set of data (e.g., use a graph, pictures, or objects). ■ Calculate the range of a data set. ■ Compare the mode and median from a set of data and determine which measure better describes the average. [RL] ■ Explain what the median represents and how to find it in a set of data. [CU] ■ Explain what the range represents and how to find it in a set of data. [CU] ■ Determine data points that would result in a given median. [RL, SP]

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

Component 1.4: Understand and apply concepts and procedures from probability and statistics.

GLE	5	6	7	8	9/10
Statistics					
1.4.3	**Understand how different collection methods or different questions can affect the results.** W ■ Ask the same question using different data collection methods that result in other points of view being supported and explain why the method affected the data. [SP, RL, CU] ■ Explain how different data collection methods affect the nature of the data set with a given question (e.g., phone survey, Internet search, person-to-person survey). [CU, MC] ■ Identify or describe the appropriate sample for a given question. ■ Identify or describe the appropriate population for a given sample	**Analyze how data collection methods affect the data collected.** W ■ Evaluate how a question or data collection method may affect the data. [RL] ■ Determine whether a sampling method will result in a representative sample. ■ Describe a data collection method that will provide an unbiased sample. [CU] ■ Compare data collection methods for a given situation to determine fairness of the method (e.g., compare a phone survey, a web survey, and a personal interview survey). [RL, MC] ■ Identify different ways of selecting a sample (e.g., convenience sampling, response to a survey, random sampling) and explain which method makes a sample more representative for a population. [SP, MC]	**Apply data collection processes to inform, persuade, or answer questions.** W ■ Formulate a question and collect data from a population, describing how the questions, collection method, and sample population affect the results. [CU] ■ Present collected data to support an opinion to inform or persuade an identified audience. [CU, MC] ■ Determine whether given data provides useful information for a situation (e.g., given a set of data, decide whether all of the information provided is necessary). [SP] ■ Determine whether data supports a given opinion and explain the decision. [CU] ■ Identify a sample relevant to a given question and population.	**Analyze how different samples of a population affect the data.** W ■ Identify sources of sampling bias given a situation (e.g., interviewing only girls, only a certain age group, or too few people). [CU, MC] ■ Describe a procedure for selecting an unbiased sample. [CU, MC] ■ Compare the results of a survey given two different sample groups. [RL, CU] ■ Identify the appropriate population for a given research question. ■ Describe how sampling may have affected the resulting data. [CU]	**Apply appropriate methods and technology to collect data or evaluate methods used by others for a given research question.** W ■ Identify sources of bias in data collection questions, samples, and/or methods and describe how such bias can be controlled. [RL, CU] ■ Evaluate methods and technology used to investigate a research question. [CU, MC] ■ Collect data using appropriate methods. ■ Use technology appropriately to collect data. [RL, MC] ■ Identify inappropriate data collection methods that might impact the accuracy of the results of a given situation. [RL, CU] ■ Determine whether the sample for a given study was from a representative sample. ■ Determine whether the methods of data collection used were appropriate for a given question or population. [RL]
1.4.4	**Understand and apply the mean of a set of data.** W ■ Explain how to find the mean of a set of data and explain the significance of the mean. [CU]. ■ Find the mean from a given set of data using objects, pictures, or formulas. ■ Given a problem situation, determine and defend whether mean, median, or mode is the most appropriate measure of average. [SP, RL, CU, MC] ■ Compare the mean, median, and mode for a given set of data. [RL] ■ Find and compare mean for two samples from the same population. [RL]	**Apply measures of central tendency to interpret a set of data.** W ■ Determine when it is appropriate to use mean, median, or mode and why a specific measure provides the most useful information in a given context. [RL, CU] ■ Use mean, median, and mode to explain familiar situations (e.g., the heights of students in the class, the hair color of students in the class). [CU, MC] ■ Find the missing number given a mean for a data set with a missing element (e.g., given a set of homework scores and the desire to earn an average score of 80%, determine what score the student must earn on the next assignment). [SP, RL]	**Understand how variations in data may affect the choice of data analysis techniques used.** W ■ Determine and use range and measures of central tendency to describe a set of data. ■ Describe the effects of extreme values on means in a population. [CU, MC] ■ Explain the difference between median or mean as a measure of central tendency in a given situation (e.g., when an extreme value skews the mean). [RL, CU, MC] ■ Describe how additional data added to data sets may affect the result of measures of central tendency. [SP, CU] ■ Find the range of a set of data. ■ Explain what the range adds to measures of central tendency. [CU]	**Analyze variations in data to determine the effect on the measures of central tendency.** W ■ Identify clusters and outliers and determine how clusters or outliers may affect measures of central tendency. [RL] ■ Alter a set of data so that the median is a more reasonable measure than the mean. [RL, CU, MC] ■ Use and interpret the most appropriate measure of central tendency and the range to describe a given set of data (e.g., the model hourly wage earned by eighth graders is $5.75 per hour and the range is $5.00 to $6.50; therefore, there are very small differences in hourly wages for eighth graders). [RL, CU, MC]	**Understand and apply techniques to find the equation for a reasonable linear model.** W ■ Determine the equation for a reasonable line to describe a set of bivariate data. [RL, MC] ■ Determine the equation of a line that fits the data displayed on a scatter plot. [SP, RL] ■ Use technology to determine the line of best fit for a set of data. [MC] ■ Match an equation with a set of data. [MC] ■ Match an equation with a graphic display. [MC] ■ Create a graph based on the equation for a line.

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.4: Understand and apply concepts and procedures from probability and statistics.

GLE	K	1	2	3	4
Statistics					
1.4.5	**Understand how a display provides information.** ■ Answer questions about graphs (e.g., how many cats? How many dogs?). [CU]	**Understand how a display provides information.** ■ Answer questions about bar graphs or pictographs (e.g., how many dancers, plants, canoes, pets?). [CU]	**Understand how a display provides information about a question.** ■ Conduct a survey for a predetermined question and collect data using tallies, charts, lists, or pictures (e.g., who has animals at home, how many, what type?). [SP, RL] ■ Identify a question that could be answered from a display. ■ Interpret results and draw conclusions from displays (e.g., pictographs, bar graphs) using comparative language (e.g., more, fewer). [CU, MC] ■ Read the labels from each axis of a graph. [CU]	**Understand representations of data from tables, charts, and bar graphs.** W ■ Pose questions that can be answered from a given graph. [CU, MC] ■ Make inferences based on the data or determine if the data can support inferences made. [CU, MC] ■ Read and report on data from tables, charts, and bar graphs. [CU] ■ Explain how types of graphs or the graph construction can support different points of view (e.g., starting the axis numbers at 50 rather than 0). [CU, SP, RL] ■ Create bar graphs including labels for title, both axes, scale units (e.g., 2s, 5s, 10s), and key if needed. [SP, RL, CU, MC] ■ Interpret graphs for comparative information (e.g., find the difference in selected data). [RL, CU, MC]	**Understand representations of data from line plots and pictographs.** W ■ Read data from line plots and pictographs. ■ Describe a trend from a given line plot. [CU, MC] ■ Interpret a pictograph where the scale is other than one unit. [RL] ■ Create two different graphic displays using a set of data. [CU, MC] ■ Read and interpret data from line plots and pictographs. [RL, CU] ■ Use technology to create pictographs. ■ Explain the data in a given table, chart, or graph. [CU] ■ Analyze the completeness and accuracy of data in a graph given a set of data. [RL]
1.4.6					

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

Component 1.4: Understand and apply concepts and procedures from probability and statistics.

GLE	5	6	7	8	9/10
Statistics					
1.4.5	**Apply strategies to organize, display, and interpret data.** W ■ Read and interpret data from text, line and bar graphs, histograms, stem-and-leaf plots, and circle graphs, and determine when using each of these is appropriate. ■ Use histograms, pictographs, and stem-and-leaf plots to display data. [CU, MC] ■ Construct assorted graphs that include labels, appropriate scale, and key. [CU] ■ Determine what type of data should be represented on a bar graph, circle graph, histogram, or line graph. [RL] ■ Compare the consistency of results from two different displays that address the same question.	**Understand how to organize, display, and interpret data in text from single line graphs and scatter plots.** W ■ Justify a choice of a graph type for a given situation using information about the type of data. [RL, CU, MC] ■ Read and interpret data from single line graphs and scatter plots, and determine when the use of these graphs is appropriate. [RL, CU] ■ Use an appropriate representation to display data (e.g., table, graphs) given a particular situation and audience. [MC, CU] ■ Make inferences based on a set of data. [RL] ■ Use data from a table, graph, or chart to support an interpretation. [RL, CU] ■ Use technology to generate bar graphs, line graphs, and scatter plots from tables of data. [MC]	**Understand and apply various data display techniques including box-and-whisker plots.** W ■ Read and interpret various data displays. ■ Determine the appropriate representation for given data. [RL, CU] ■ Construct bar graphs, circle graphs, line graphs, box-and-whisker and scatter plots using collected data. [CU, MC] ■ Use scatter plots to describe trends and interpret relationships. [RL, CU] ■ Read and interpret data from box-and-whisker plots and determine when using this type of graph is appropriate. [RL, CU] ■ Describe statistical information given a box-and-whisker plot (e.g., median, range, interquartile range). [CU] ■ Compare different graphical representations of the same data. [RL, MC] ■ Make and justify an inference drawn from a sample. [RL, CU, MC]	**Understand and apply data techniques to interpret bivariate data.** W ■ Interpret graphic and tabular representations of bivariate data. ■ Use a line of best fit to predict a future value of a variable. [RL] ■ Use a line of best fit to interpolate between existing data values. [RL] ■ Draw trend lines with or without technology and make predictions about real-world situations (e.g., population trends, socio-economic trends). [CU, MC, RL] ■ Examine data in a two-column table to interpolate or extrapolate additional values. [RL] ■ Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken (e.g., age groups, regions of the U.S., genders, racial/ethnic distributions). [RL, MC, CU]	**Analyze a linear model to judge its appropriateness for a data set.** W ■ Determine whether a straight line is an appropriate way to describe a trend in a set of bivariate data. [MC, RL] ■ Determine whether the underlying model for a set of data is linear. [RL, MC] ■ Decide and explain whether it is appropriate to extend a given data set following a line of best fit. [RL, MC] ■ Determine whether a linear prediction from a given set of data is appropriate for the data and support the decision with data. [MC]. ■ Determine whether an equation for a line is appropriate for a given set of data and support the judgment with data. [RL, MC] ■ Use technology to generate data to fit a linear model. [SP, MC]
1.4.6		**Evaluate a data set to determine how it can be, or has been, used to support a point of view.** W ■ Compare graphs to data sets (e.g., given unlabeled graphs and data sets, match the appropriate data to a graph). [RL] ■ Judge the appropriateness of inferences made from a set of data and support the judgment. [CU, MC] ■ Identify claims based on statistical data and assess the validity of the claims. [CU, RL] ■ Explain whether the scale on a graph accurately represents the data. [CU] ■ Compare or evaluate two or more interpretations of the same set of data for accuracy.	**Evaluate how different representations of the same set of data can support different points of view.** W ■ Critique the use of data and data displays for univariate data. ■ Judge the reasonableness of conclusions drawn from a set of data and support that position with evidence (e.g., from newspapers, Web sites, opinion polls). [MC, RL] ■ Determine the accuracy and completeness of the data in a table or graph. [RL, CU] ■ Explain how different representations of the same set of data can support different points of view. [RL, CU] ■ Describe how statistics or graphics have been used or misused to support a point of view.	**Evaluate how statistics and graphic displays can be used to support different points of view.** W ■ Critique the use of data and data displays for bivariate data. [RL] ■ Judge the reasonableness of conclusions drawn from a set of data and support that position with evidence (e.g., from newspapers, Web sites, opinion polls). [MC, RL] ■ Determine whether a prediction is reasonable based on a trend line and explain the rationale. [RL] ■ Determine whether claims made about results are based on biased representations of data (e.g., whether a scale has been intentionally used to support a point of view).	**Apply understanding of statistics to make, analyze, or evaluate a statistical argument.** W ■ Identify trends in a set of data in order to make a prediction based on the information. [CU, MC] ■ Justify a prediction or an inference based on a set of data. [CU, MC] ■ State possible factors that may influence a trend but not be reflected in the data (e.g., population growth of deer vs. availability of natural resources or hunting permits). [MC, CU, RL] ■ Use statistics to support different points of view. [RL] ■ Analyze a set of statistics to develop a logical point of view. [RL. CU, MC] ■ Justify or refute claims and supporting arguments based on data. [CU, MC] ■ Determine whether statistics have been used or misused to support a point of view or argument and support the evaluation with data. [RL]

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.5: Understand and apply concepts and procedures from algebraic sense.

GLE	K	1	2	3	4
Patterns, functions, and other relations					
1.5.1	**Know how to recognize patterns.** ■ Identify and extend patterns (e.g., ABAB, green-green-blue, counting). [RL] ■ Create an AB pattern.	**Understand the concept of patterns.** ■ Create and describe a variety of repeating patterns using sounds, objects, and symbols. [CU] ■ Describe and extend a repeating pattern (e.g., ABAC, ABAC; snap, clap, snap, stomp). [CU] ■ Identify the unit in a repeating pattern (e.g., in A-A-B-A-A-B the unit is A-A-B). [RL] ■ Identify and describe numerical patterns in the 100s chart. [CU, RL] ■ Identify geometric patterns in art, textiles, and ceramics.	**Understand how patterns are generated.** ■ Translate a pattern from one representation to another (e.g., snap-clap-stomp translates to ABC). [CU, MC] ■ Identify, extend, create, and explain patterns of addition and subtraction represented in charts and tables. [CU, RL, MC]	**Understand patterns of objects including number patterns with a single addition or subtraction operation.** W ■ Recognize and extend patterns of numbers, figures, and objects using addition and subtraction based on a single arithmetic operation between the terms (e.g., stacking cans in a pyramid, observing textile patterns). ■ Identify, extend, and describe numerical patterns (e.g., skip counting, 100 chart, multiplication table). [RL, CU] ■ Describe the pattern in a number sequence (e.g., Guess My Rule, Function Machine). [CU] ■ Identify the rule for a pattern based on a single operation (e.g., add 3). [RL] ■ Explain what makes a given pattern a pattern. [CU] ■ Complete a pattern by supplying missing elements in the pattern. ■ Compare two patterns to determine whether they are alike or different and explain the decision. [RL, CU]	**Understand patterns of objects including number patterns using addition, subtraction, or multiplication based on a single arithmetic operation.** W ■ Extend or create patterns of numbers, shapes, or objects using addition, subtraction, or multiplication based on a single operation between terms. ■ Extend and represent patterns using words, tables, numbers, and pictures. [RL, CU] ■ Create a number pattern and explain what makes it a pattern. [CU]
1.5.2					**Understand a pattern to develop a rule describing the pattern which may include a single arithmetic operation.** W ■ Use the rule for a pattern which may include a single arithmetic operation to extend or fill in parts of a pattern. ■ Solve a problem that uses a pattern with a single operation. [SP] ■ Model growing patterns using objects and pictures (e.g., a stair step sequence, or a "growing" L shape in which a unit is added to each leg to show 3, 5, 7, 9, . . .). [RL, CU] ■ Describe the rule for a pattern based on one operation (e.g., add 4; multiply by 2). [CU] ■ Analyze a pattern to determine a rule. [RL] ■ Use a rule to generate a pattern.

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

Component 1.5: Understand and apply concepts and procedures from algebraic sense.

GLE	5	6	7	8	9/10
Patterns, functions, and other relations					
1.5.1	**Understand patterns of objects including relationships between two sets of numbers based on a single arithmetic operation.** W ■ Extend or create patterns of numbers, shapes, or objects based on a single arithmetic operation between the terms. ■ Determine the operation that changes the elements of one set of numbers into the elements of another set of numbers (e.g., if one set is 1,2,3, … and another set is 5,10, 15 … , one rule is to multiply each number in the first set by 5 to get the corresponding number in the second set). [RL] ■ Explain why a given rule fits a pattern based on a single arithmetic operation in the rule. [RL, CU]	**Apply rules for number patterns based on two arithmetic operations.** W ■ Recognize or extend patterns and sequences using operations that alternate between terms. [RL] ■ Create, explain, or extend number patterns involving two related sets of numbers and two operations including addition, subtraction, multiplication, or division. [CU] ■ Use rules for generating number patterns (e.g., Fibonacci sequence, bouncing ball) to model real-life situations. [MC] ■ Use technology to generate patterns based on two arithmetic operations. [SP] ■ Supply missing elements in a pattern based on two operations. ■ Select or create a pattern that is equivalent to a given pattern.	**Apply understanding of linear relationships to analyze patterns, sequences, and situations.** W ■ Identify patterns that are linear relations and provide missing terms. [RL] ■ Describe the relationship between the terms in a sequence and their positions in the sequence. [CU] ■ Identify, extend, or represent patterns and sequences using tables, graphs, or expressions. [RL, MC] ■ Use technology to generate graphic representations of linear relationships. [SP] ■ Make predictions using linear relationships in situations. [RL] ■ Identify a linear relationship that has the same pattern as another linear relationship. ■ Create a representation of a linear relationship given a rule. [MC]	**Apply understanding of linear and non-linear relationships to analyze patterns, sequences, and situations.** W ■ Extend, represent, or create linear and non-linear patterns and sequences using tables and graphs. [RL] ■ Explain the difference between linear and non-linear relationships. [CU] ■ Predict an outcome given a linear relationship (e.g., from a graph of profit projections, predict the profit). [RL] ■ Use technology to generate linear and non-linear relationship. [SP, RL]	**Apply processes that use repeated addition (linear) or repeated multiplication (exponential).** W ■ Recognize, extend, or create a pattern or sequence between sets of numbers and/or linear patterns. [RL, CU, MC] ■ Identify, extend, or create a geometric or arithmetic sequence or pattern. [RL, CU] ■ Translate among equivalent numerical, graphical, and algebraic forms of a linear function. [RL, MC] ■ Make predictions based on a pattern or sequence.
1.5.2	**Apply understanding of a pattern to develop a rule describing the pattern including combinations of two arithmetic operations.** W ■ Use the rule for a pattern which may include a combination of two arithmetic operations to extend a pattern. [SP, RL] ■ Solve a problem that uses a pattern of alternating operations (e.g., a frog climbed up 3 feet each day and then slipped down 1 foot each night, how long did it take the frog to reach the top of a building that is 15 feet high?). [SP] ■ Analyze a pattern to determine a rule with two operations between terms. [RL] ■ Use a rule to generate a pattern.	**Apply understanding of patterns involving two arithmetic operations to develop a rule.** W ■ Describe the rule for a pattern with combinations of two arithmetic operations in the rule. ■ Identify patterns involving combinations of operations in the rule, including exponents (e.g., 2, 5, 11, 23). [RL, MC] ■ Represent a situation with a rule involving a single operation (e.g., presidential elections occur every four years; when will the next three elections occur after a given year?). [CU, MC] ■ Create a pattern involving two operations using a given rule.	**Apply understanding of linear patterns in a table, graph, or situation to develop a rule.** W ■ Describe the rule and/or construct a table to represent a pattern with combinations of two arithmetic operations in the rule. ■ Write an expression or equation with a single variable representing a situation or real-world problem. [CU, MC] ■ Write a story about a situation that represents a given linear equation, expression, or graph. [CU, MC] ■ Describe the rule or construct a table to represent a pattern with combinations of two arithmetic operations in the rule. [RL, CU] ■ Use technology to determine the rule for a linear relationship. [SP, RL]	**Analyze a pattern, table, graph, or situation to develop a rule.** W ■ Use technology to help develop a table or graph from an iterative definition (e.g., the number of cells doubles every hour starting with one cell at noon). [CU, MC] ■ Explain the nature of changes in quantities in linear relationships using graphs, tables, or expressions. [CU, MC] ■ Develop recursive equations that describe linear relations in terms of current and previous values (e.g., start = 7; Current = Previous + 5 would give a set of values (1,7),(2,12), (3,17) …). ■ Use words or algebraic symbols to describe a rule for a linear relationship between two sets of numbers (e.g., given a table, describe a rule). [CU]	**Analyze a pattern, table, graph, or model involving repeated addition (linear) or repeated multiplication (exponential) to write an equation or rule.** W ■ Find the equation of a line in a variety of ways (e.g., from a table, graph, slope-intercept, point-slope, two points). [RL, MC] ■ Generate and use rules for a pattern to make predictions about future events (e.g., population growth, future sales, growth of corn stalks, future value of savings account). [SP, RL, MC] ■ Identify or write an equation or rule to describe a pattern, sequence, and/or a linear function. [RL, CU, MC] ■ Write an equation for a line given a set of information (e.g., two points, point-slope, etc.). [CU, MC] ■ Write a recursive definition of a geometric pattern (e.g., Start and New = Old * Number). [CU, MC] ■ Represent systems of equations and inequalities graphically. [RL, MC] ■ Write a story that represents a given linear equation or expression. [CU, MC] ■ Write an expression, equation, or inequality with two variables representing a linear model of a real-world problem. [CU, MC]

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.5: Understand and apply concepts and procedures from algebraic sense.

GLE	K	1	2	3	4
Symbols and representations					
1.5.3	**Understand the concepts of equality and inequality.** ■ Use physical objects to model language (e.g., same, different, equal, not equal, more, less). [CU] ■ Model/act out story problems to solve whole number equations and inequalities (e.g., there are three kids; two have three crayons and one has two crayons. How can you make it so all kids have the same number of crayons?). [CU, MC]	**Understand the meaning of symbols and labels used to represent equality in situations.** ■ Demonstrate equality by recording number sentences with balance using the "=" symbol (e.g., 9 = 4 + 5, 4 + 5 = 2 + 7, 9 = 9). [CU] ■ Complete open sentences showing equalities (e.g., 5 = _____). ■ Explain, using pictures or words, the meaning of equality. [CU] ■ Give an example of equality in real life (e.g., on the first turn, Juan scored 4 points; on the second turn, he scored 5 points. On the first turn, Ivana scored 2 points; on the second turn, she scored 7 points. After two turns, they are tied with the same number of points.). [MC]	**Understand the meaning of symbols and labels used to represent situations.** ■ Use number sentences with symbols and labels to represent real-world problems involving addition and subtraction. [MC] ■ Give an example of inequality in real life (e.g., on the first turn, Juan scored 6 points; on the second turn, he scored 8 points. On the first turn, Ivana scored 9 points; on the second turn, she scored 7 points. After two turns, Juan's points are less than Ivana's points.). [CU, MC]	**Apply understanding of the concept of mathematical equality.** W ■ Write an equation or expression for a given situation (e.g., there are 23 dogs at a kennel, if 15 are present, how many are absent?). [SP, RL, CU] ■ Explain equality and the use of "=" in equations. [CU] ■ Compare expressions to determine whether they are equal (e.g., 3 + 4 and 2 + 5). [RL] ■ Write a situation that represents a given equation involving addition or subtraction. [CU, MC] ■ Identify a situation that represents a given equation involving addition or subtraction. [CU, MC]	**Apply understanding of the concept of mathematical inequality.** W ■ Compare multiplication or division expressions using the symbols >, <, and = (e.g., 5 x 3 > 3 x 2). [RL] ■ Select operational and relational symbols to make a multiplication or division number sentence true (e.g., 4 _ 3 = 12; 5 x 12 _ 64). ■ Explain inequality and the use of ">" or "<" in inequalities. [CU] ■ Identify or write a situation that represents it given an expression or equation using < or >. [CU, MC]
1.5.4				**Understand and apply operational and relational symbols and notations to write equations involving addition and subtraction.** W ■ Write and explain mathematical statements (e.g., 7 + ❑ = 8 or ❑ +8 = 10). [CU] ■ Identify and use mathematical symbols and notations in reading and writing expressions and equations involving addition and subtraction. ■ Write an equation for a given situation (e.g., there are 23 children in a class; if 15 are present, how many are absent?). [CU]	**Understand and apply operational and relational symbols and notations to write expressions and equations involving multiplication and division.** W ■ Identify and use mathematical symbols and notations in reading and writing expressions and equations. ■ Write a situation that represents a given equation involving multiplication or division. [CU, MC] ■ Write an equation that represents a given situation involving multiplication or division. [CU, MC]

SP: Solves Problems RL: Reasons Logically CU: Communicates Understanding MC: Makes Connections

Component 1.5: Understand and apply concepts and procedures from algebraic sense.

GLE	5	6	7	8	9/10
Symbols and representations					
1.5.3	**Apply understanding of the concept of mathematical inequality.** W ■ Express relationships between quantities using " ≠, ≤, or ≥". ■ Given a number sentence using ≠, ≤, or ≥, identify or write a situation that represents it. [CU, MC] ■ Given a real-world situation, use =, ≠, ≤, or ≥ to describe quantities. [RL, MC] ■ Explain inequality and the use of "≠", "≤", or "≥". [CU]	**Apply understanding of equalities and inequalities to interpret and represent relationships between quantities.** W ■ Express relationships between quantities (decimals, percents, and integers) using =, ≠, <, >, ≤, and ≥. [CU] ■ Match a given situation to the correct inequality or equality. [MC] ■ Express relationships between non-negative rational numbers using symbols. ■ Write an inequality with a single variable to match a particular situation. [RL, CU]	**Understand relationships between quantities using squares and square roots.** W ■ Represent relationships between quantities using exponents (squares) and radicals (roots). [CU] ■ Simplify square roots of square numbers (e.g., the square root of 9 is 3). [RL] ■ Demonstrate understanding of square roots with physical models and examples. [CU] ■ Use exponents (squares) and radicals (square roots) to represent relationships (e.g., finding the area of a square with a side of 5 could be represented by 5^2). [CU]	**Understand relationships between quantities including whole number exponents, square roots, and absolute value.** W ■ Represent relationships between quantities using exponents (squares) and radicals (roots). [CU] ■ Explain the placement of numbers including square roots and exponents on a number line. [CU] ■ Model or describe a real-life situation using absolute value (e.g., the taxi-cab distance from one point to another can be represented by the sum of two absolute values). [CU, MC] ■ Use relational symbols to express relationships between rational numbers including percents, square roots, absolute value, and exponents. [CU]	
1.5.4	**Understand how to represent situations involving one operation or two alternating arithmetic operations.** W ■ Translate a situation involving one arithmetic operation into algebraic form using equations, tables, and graphs. [CU, MC] ■ Translate a situation involving two alternating arithmetic operations into algebraic form using equations, tables, and graphs (e.g., a snail crawls up 3 feet each day and slides back 2 feet each night). [CU, MC] ■ Identify or describe a situation involving one arithmetic operation that may be modeled by a graph. [CU] ■ Identify or describe a situation involving two alternating arithmetic operations that may be modeled by a graph (e.g., a snail crawls up 3 feet each day and slides back 2 feet each night). [CU]	**Apply understanding of tables, graphs, expressions, equations, or inequalities to represent situations involving two arithmetic operations.** W ■ Translate a situation involving multiple arithmetic operations into algebraic form using equations, tables, and graphs. [RL, CU, MC] ■ Identify or describe a situation involving two arithmetic operations that matches a given graph. [CU, MC] ■ Represent an equation, expression, or inequality using a variable in place of an unknown number. [CU] ■ Represent or evaluate algebraic expressions involving a single variable. [RL, CU] ■ Represent an equation or expression using a variable in place of an unknown number. [RL, CU] ■ Identify a situation that corresponds to a given equation or expression.	**Apply understanding of equations, tables, and graphs to represent situations involving linear relationships.** W ■ Represent linear relationships through expressions, equations, tables, and graphs of situations involving non-negative rational numbers. ■ Graph data to demonstrate relationships in familiar contexts (e.g., conversions, perimeter, area, volume, and scaling). [CU, MC] ■ Develop a situation that corresponds to a given equation or expression. [CU, MC] ■ Create a table or graph given a description of, or an equation for, a situation involving a linear relationship. [CU, MC] ■ Describe a situation involving a linear or non-linear relationship that matches a given graph (e.g., time-distance, time-height). [CU, MC] ■ Explain the meaning of a variable in a formula, expression, or equation. [CU]	**Apply understanding of concepts of algebra to represent situations involving single-variable relationships.** W ■ Represent variable quantities through expressions, linear equations, inequalities, tables, and graphs of situations. [CU] ■ Write an expression, equation, or inequality with a single variable representing a situation or real-world problem. [SP, RL, MC] ■ Identify and use variables to read and write relationships involving rational numbers. ■ Model a given description or situation involving relationships with a graph or table. [CU, MC] ■ Describe a situation involving relationships that matches a given graph. [CU, MC] ■ Create a table or graph given a description of, or an expression for, a situation involving a linear or non-linear relationship. [CU, MC]	**Apply understanding of equations, tables, or graphs to represent situations involving relationships that can be written as repeated addition (linear) or repeated multiplication (exponential).** W ■ Represent variable quantities through expressions, equations, inequalities, graphs, and tables to represent linear situations involving whole number powers and square and cube roots. [CU, MC] ■ Identify and use variable quantities to read and write expressions and equations to represent situations that can be described using repeated addition (e.g., models that are linear in nature). [CU, MC] ■ Identify and use variable quantities to read and write expressions and equations to represent situations that can be described using repeated multiplication (e.g., models that are exponential such as savings accounts and early stages of population growth). [CU, MC] ■ Recognize and write equations in recursive form for additive models (e.g., starting value, New = Old + some number). [CU, MC] ■ Recognize and write equations in recursive form for multiplicative models (e.g., starting value, New = Old x some number). [CU, MC] ■ Select an expression or equation to represent a given real world situation. [MC]

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.5: Understand and apply concepts and procedures from algebraic sense.

GLE	K	1	2	3	4
Evaluating and solving					
1.5.5					**Understand and apply a variety of strategies to evaluate expressions with addition, subtraction, or multiplication.** W ■ Substitute a numeric value for a symbol in expressions or equations (e.g., if ❑ = 7, find ❑ x 3; if *w* = 12 and *l* = 36, what is *w* x *l*?).
1.5.6			**Understand and apply strategies to solve for the unknown using addition and subtraction.** ■ Solve equations with an "unknown" (e.g., 6 + ❑ = 11; 11 = ❑ +6). [RL] ■ Justify the selection of a particular value for an unknown quantity in a real world situation (e.g., two girls had 10 cookies. If Kwame had 6, how many did Ellie have? Explain). [RL, MC]	**Understand and apply strategies to solve equations that include addition or subtraction.** W ■ Solve problems involving equality (e.g., 5 + 3 = ❑ + 2). [SP, RL] ■ Solve equations with addition and subtraction using manipulatives, pictures, and symbols. [SP, RL, CU] ■ Describe a strategy used to solve an equation with addition or subtraction. [CU]	**Understand and apply strategies to solve equations that include multiplication.** W ■ Solve missing factor equations (e.g., ❑ x 3 = 12). [SP, RL] ■ Describe and compare strategies used to solve an equation with multiplication. [SP, RL, CU]

SP: Solves Problems **RL**: Reasons Logically **CU**: Communicates Understanding **MC**: Makes Connections

Component 1.5: Understand and apply concepts and procedures from algebraic sense.

GLE	5	6	7	8	9/10
Evaluating and solving					
1.5.5	**Understand and apply a variety of strategies to evaluate expressions with division.** W ■ Evaluate expressions with division using manipulatives, pictures, and symbols. ■ Substitute a symbol for a numeric value in an expression (e.g., ✱ = 4, find 20 ÷ ✱; if ★ = 12 and ▼ = 36, what is ▼ ÷ ★?). [SP, RL]	**Understand and apply procedures to evaluate expressions and formulas.** W ■ Evaluate simple expressions and formulas using pictures and/or symbols. [RL] ■ Represent and evaluate algebraic expressions involving a single variable. [RL, CU] ■ Evaluate an expression by substituting non-negative values for variables (e.g., find the value of $3y + 2$ when $y = 3$). [RL, MC] ■ Determine the expression that represents a given situation. [MC, CU] ■ Describe a situation that fits with a given expression. [RL, MC, CU]	**Understand and apply procedures to evaluate expressions and formulas considering order of operations.** W ■ Substitute non-negative rational values for variables in order to evaluate expressions and formulas (e.g., length x width when length = 3 and width = 4) ■ Explain the simplification of expressions and equations using order of operations. [CU] ■ Evaluate expressions and formulas considering order of operations. [RL] ■ Determine the expression that represents a given situation. [MC, CU] ■ Describe a situation that fits with a given expression. [RL, MC, CU] ■ Write expressions or equations for a situation.	**Understand and apply the procedures for simplifying single-variable expressions.** W ■ Simplify expressions and evaluate formulas involving integers. [RL, MC] ■ Match expressions to equivalent simplified expressions. [MC] ■ Explain a simplification of an expression involving integers. [CU] ■ Simplify expressions by combining like terms. ■ Simplify expressions using mathematical properties (distributive, commutative, associative, etc.). [RL] ■ Determine the expression that represents a given situation. [MC, CU] ■ Describe a situation that fits with a given expression. [RL, MC, CU]	**Apply procedures to simplify expressions.** W ■ Simplify expressions and evaluate formulas involving exponents. ■ Justify a simplification of an expression involving exponents. [RL, CU] ■ Use multiple mathematical strategies and properties to simplify expressions.
1.5.6	**Understand and apply strategies to solve equations that include division.** W ■ Solve for a missing value in an equation involving division (e.g., 12 ÷ ❑ = 3). [SP, RL] ■ Describe and compare strategies used to solve an equation with multiplication or division. [SP, RL, CU]	**Understand and apply a variety of strategies to solve one-step equations.** W ■ Solve one-step equations using pictures and symbols. ■ Solve one-step single variable equations using any strategy (e.g., what number goes in the mystery box?). ■ Solve real-world situations involving single variable equations. [CU, MC] ■ Explain a strategy for solving a single variable equation. [CU] ■ Write and solve one-step single variable equations for a given situation. [MC]	**Understand and apply a variety of strategies to solve two-step equations with one variable.** W ■ Explain and justify the solution to a problem in a given context. [RL, CU, MC] ■ Solve two-step equations with one variable on only one side of the equal sign (e.g., $2x + 4 = 12$).	**Understand and apply a variety of strategies to solve multi-step equations and one-step inequalities with one variable.** W ■ Solve multi-step equations and one-step inequalities with one variable. ■ Solve single variable equations involving parentheses, like terms, or variables on both sides of the equal sign. ■ Solve one-step inequalities (e.g., $2x < 6$, $x + 4 > 10$). ■ Solve real-world situations involving single variable equations and proportional relationships and verify that the solution is reasonable for the problem. [SP, RL, CU]	**Apply procedures to solve equations and systems of equations.** W ■ Rearrange formulas to solve for a particular variable (e.g., given $A = .5bh$, solve for h). [MC, CU] ■ Solve real-world situations involving linear relationships and verify that the solution makes sense in relation to the problem. [SP, RL, CU, MC] ■ Find the solution to a system of linear equations using tables, graphs, and symbols. [CU, MC] ■ Interpret solutions of systems of equations. [CU, MC] ■ Solve multi-step equations. [SP, RL] ■ Use systems of equations to analyze and solve real-life problems. [SP, CU, MC] ■ Determine when two linear options yield the same outcome (e.g., given two different investment or profit options, determine when both options will yield the same result). ■ Use systems of equations to determine the most advantageous outcome given a situation (e.g., given two investment options, determine under what conditions each will yield the best result). [MC, SP]

EALR 2: The student uses mathematics to define and solve problems.

Component 2.1: Understand problems.

GLE	K	1	2	3	4
	Example: A classroom needs a playground ball for each student in the class. The class has fewer playground balls than are needed.	**Example:** A classroom is presenting a play and everyone has invited two guests. Enough chairs are needed to seat all the guests. There are some chairs in the classroom.	**Example:** A classroom is planning an all-day skating party on Thursday. Each student must pay for admission ($2), a box lunch ($3), and skate rental ($2). The teacher needs a total amount to reserve the rink.	**Example:** Miguel's reading class has set a goal to increase nightly reading to at least 30 minutes. He is taking a survey of his nine classmates to determine about how many minutes they read each night to see if they have met the goal. Miguel likes to read books by Matt Christopher.	**Example:** Jamal and his sister, Aleesha, want to buy a pet. Their mother said she will help by paying for the ongoing cost of food if they can save the money to buy the pet and all the needed equipment and food to get started. They have $17.83 saved already and most of that money is in quarters. They are reading pet store ads to see what the costs would be if they bought a mouse, a hamster, or a guinea pig.
2.1.1	**Understand how to define a problem in a familiar situation with teacher guidance.** ■ State information presented in teacher-led discussion to determine if there is a problem that needs an answer (e.g., a classroom activity requires a playground ball for each student. There are some balls available in the classroom). ■ State the problem in own words (e.g., are there enough playground balls? If not, how do we get enough for the class?). ■ Generate questions that would need to be answered in order to solve the problem (e.g., how many balls are in the classroom? How many more do we need?). ■ Identify known and unknown information with teacher guidance (e.g., known — the number of students in the class, and the number of balls needed; unknown — the number of additional playground balls needed). [1.1.5]	**Understand how to define a problem in a familiar situation with teacher guidance.** ■ State information presented in a teacher-led discussion to determine if there is a problem (e.g., a classroom is having a play and each student invited two guests. Chairs are needed for the guests. There are some chairs available in the classroom). ■ State the problem in own words (e.g., there aren't enough chairs for the guests. How many more chairs do we need?). ■ Generate questions that would need to be answered in order to solve the problem (e.g., how many guests are attending? How many more chairs do we need?). ■ Identify known and unknown information with teacher guidance (e.g., known — number of students, number of guests invited, number of chairs in classroom; unknown — number of guests attending, number of chairs needed). [1.1.5]	**Understand how to define a problem in a familiar situation.** ■ State or record information presented in situation (e.g., the classroom is planning a skating party on Thursday. Each student must pay for admission, lunch, and skates. The teacher needs to know the total cost in order to reserve the rink). ■ Explain the problem, verbally or in writing, in own words (e.g., how much will the skating party cost?). ■ Generate questions that would need to be answered in order to solve problem (e.g., what is the cost of a ticket and skate rental for the skating rink? What is the cost of food? What is the cost for each student? What will a skating party cost?). [1.4.4] ■ Identify known and unknown information (e.g., known — the cost of admission, skates, lunch, and the number of students going; unknown — cost for each student and total cost). ■ Identify extraneous information (e.g., the party is planned for Thursday).	**Analyze a situation to define a problem.** W ■ Use strategies/approaches to examine the situation and determine if there is a problem to solve (e.g., ask questions, or paraphrase information provided: Miguel is taking a survey to determine about how many minutes students read on school nights. The class goal is at least 30 minutes each night). ■ Determine the problem using information from investigation (e.g., has the class met its reading goal for the week?). ■ Generate questions that would need to be answered in order to solve the problem (e.g., about how many minutes did each person read? Can we estimate or do we need an exact number? What is the difference between the goal and the minutes read?). ■ Identify known and unknown information (e.g., known—who the students are, the class goal [30 minutes x 5 nights x 10 students is 1500 total minutes]; unknown—the number of minutes each student read, if the class reached the goal). ■ Identify information that is needed and not needed to solve the problem (e.g., needed—the class goal; not needed—Miguel likes Matt Christopher books).	**Analyze a situation to define a problem.** W ■ Use strategies/approaches to examine the situation and determine if there is a problem to solve (e.g., ask questions, make lists, or paraphrase information provided in ads: two kids want to buy a pet. They have some money but they need to find out if they can afford a mouse, hamster, or guinea pig and the equipment and food for it). ■ Determine the problem using information from investigation (e.g., do Jamal and Aleesha have enough money?). ■ Generate questions that would need to be answered in order to solve the problem (e.g., how much will each animal cost? How much is equipment and food for each animal?). ■ Identify known and unknown information (e.g., known—how much money Jamal and Aleesha have; unknown—all the costs for each animal). ■ Identify information that is needed or not needed (e.g., needed—all costs related to purchasing the animals, the amount that the kids have saved; not needed—the money is in quarters).

GLE	5	6	7	8	9/10
	Example: Mrs. Allen's class won a pizza party sponsored by the PTA for best school attendance. There are 30 students in the class. Ten pizzas arrived but they were cut in three different ways. Three pizzas were cut in eighths, three were cut in fourths, and four were cut in halves. Mrs. Allen wouldn't let the students start eating until she was sure everyone received equal shares.	**Example:** A gardener living in Yakima has 100 feet of fencing material. Find the dimensions of the largest rectangular area that he could enclose using all of the fencing material.	**Example:** On the playground, Juan made 13 free throws out of 18 tries. If Bonita shoots 25 free throws, what is the lowest number she has to make in order to have a better free throw percentage than Juan?	**Example:** The following information was provided to a group of students. They were asked to interpret this information for someone who has a speed of 19 feet per second and also for someone who takes 5 steps per second. How would you answer these questions? Speed (ft/s) / Steps per second 15.86 / 3.05 16.88 / 3.12 17.50 / 3.17 18.62 / 3.25 19.97 / 3.36 21.06 / 3.46 22.11 / 3.55	**Example:** The following are the times (in seconds) of the Olympics in the given years. Using this information, is it reasonable to believe that the women will run as fast as the men in this event? Justify your answer using this data: Year / Men's / Women's 1948 / 10.3 / 11.9 1952 / 10.4 / 11.5 1956 / 10.5 / 11.5 1960 / 10.2 / 11.0 1964 / 10.0 / 11.4 1968 / 9.95 / 11.0 1972 / 10.14 / 11.07 1976 / 10.06 / 11.08 1980 / 10.25 / 11.06 1984 / 9.99 / 10.97 1988 / 9.92 / 10.54 1992 / 9.96 / 10.82 1996 / 9.84 / 10.94 2000 / 9.87 / 10.75
2.1.1	**Analyze a situation to define a problem.** W ■ Use strategies/approaches to examine the situation and determine if there is a problem to solve (e.g., draw pictures, ask questions, or paraphrase information provided: 30 students in a class have ten pizzas to divide fairly. Three are sliced in eighths, three are sliced in fourths and four are sliced in halves). ■ Generate questions that would need to be answered in order to solve the problem (e.g., how should the pizzas be sliced? Can we use the slices that have already been made? How many pieces is each student's fair share?). ■ Identify known and unknown information (known—number of students, number of pizzas to share,the ways in which the pizzas have been sliced; unknown—size of each slice, number of equal slices, number of pieces per student). ■ Identify information that is needed or not needed (e.g., needed—number of students, number of pizzas, how pieces have already been cut; not needed—reason for the pizza party).	**Analyze a situation to define a problem.** W ■ Use strategies to become informed about the situation (e.g., listing information, asking questions). ■ Summarize the situation (e.g., there is 100 feet of fencing and we want to enclose as much land, in the shape of a rectangle, as possible). ■ Determine whether enough information is given to find a solution (e.g., list what is needed to find the area of a rectangle and compare to the list of known things). ■ Determine whether information is missing or extraneous (e.g., compare the list of known things to the list of needed things to see if there are things that are not needed). ■ Define the problem (e.g., find the rectangle with largest area with a perimeter of 100 feet).	**Analyze a situation to define a problem.** W ■ Use strategies to become informed about the situation (e.g., listing information, asking questions). ■ Summarize the situation (e.g., two people are shooting free throws, one shot 18, the other 25; we are trying to find the percentage made for each). ■ Determine whether enough information is given to find a solution (e.g., list what is needed to find the percentage of free throws made). ■ Determine whether information is missing or extraneous (e.g., compare the list of known things to the list of needed things to see if there are things that are not needed — names, location). ■ Define the problem (e.g., find the smallest number of free throws Bonita needs to make out of 25 attempts in order to top Juan's percentage).	**Analyze a situation to define a problem.** W ■ Use strategies to become informed about the situation (e.g., listing information, asking questions). ■ Summarize the problem (e.g., we have information about the relationship between the number of steps per second and the speed in feet per second; we wish to find approximate speed or stride rates). ■ Determine whether enough information is given to find a solution (e.g., list what is needed to find the relationship between stride rate and speed; list known and unknown information). ■ Determine whether information is missing or extraneous (e.g., compare the list of known things to the list of needed things to see if there are things that are not needed — names, location). ■ Define the problem (e.g., find the relationship between the steps per second and speed).	**Analyze a situation to define a problem.** W ■ Use strategies to become informed about the situation (e.g., listing information; examine the table for patterns; create a scatter plot to look for patterns; asking questions). ■ Summarize the problem (e.g., there are Olympic winning times over the past 50 years; both men's and women's times are decreasing; will there come a time when women run faster than men). ■ Determine whether enough information is given to find a solution (e.g., list what is needed to be found; extend the pattern to see if women's times will be less). ■ Determine whether information is missing or extraneous (e.g., compare the list of known things to the list of needed things to see if there are things that are not needed). ■ Define the problem (e.g., if the pattern continues in the same fashion, will women run faster than men and, if so, when will that occur).

EALR 2: The student uses mathematics to define and solve problems.

Component 2.2: Apply strategies to construct solutions.

GLE	K	1	2	3	4
2.2.1	**Understand how to create a plan to solve a problem with teacher guidance.** ■ Gather and organize categorical data (e.g., in a teacher-led activity, create a two-column chart — one column for student names, and tally marks in the other to represent which students are assigned a ball). [1.4.3]	**Understand how to create a plan to solve a problem with teacher guidance.** ■ Gather and organize categorical data (e.g., in a teacher-guided activity, create a two-column chart — one column for student names and the other to record the number of guests attending the play). [1.4.3]	**Understand how to create a plan to solve a problem.** ■ Gather and organize relevant information (e.g., create a four-column chart with student names in one column and the other three for costs related to the party — admission, skates, lunch; draw a seating chart and write in costs by each student).	**Apply strategies, concepts, and procedures to devise a plan to solve the problem.** W ■ Gather and organize data and information (e.g., create a survey to find out about how many minutes students are watching TV; organize data on a two-column chart). ■ Determine what strategy will be used to solve the problem (e.g., estimate minutes read per night per week from data gathered).	**Apply strategies, concepts, and procedures to devise a plan to solve the problem.** W ■ Gather and organize data (e.g., determine how to break information into categories such as cost of animal, cost of cage, cost of food, cost of bedding, cost of equipment in order to create a table). ■ Determine what tools should be used to construct a solution (e.g., paper and pencil, calculator, mental math, physical models such as play money).
2.2.2	**Apply mathematical tools to solve the problem with teacher guidance.** ■ Use appropriate tools to find a solution (e.g., draw pictures, use chart to count how many empty spaces there are for the playground balls). [1.1.1, 1.1.5] ■ Recognize when an approach is unproductive and try a new approach.	**Apply mathematical tools to solve the problem with teacher guidance.** ■ Use strategies (chart to count, skip count, cluster, or physical models). [1.1.1, 1.1.5] ■ Use appropriate tools from among mental math, paper and pencil, manipulatives, or calculator (e.g., to determine the total number of guests attending and the total number of chairs needed for the class play). [1.1.7] ■ Recognize when an approach is unproductive and try a new approach.	**Apply mathematical tools to solve the problem.** ■ Use estimation strategies (e.g., front-end estimation, clustering) to predict computation results. [1.1.8] ■ Use appropriate tools from among mental math, paper and pencil, manipulative, or calculator (e.g., to determine the total cost of the skating party). [1.1.7] ■ Recognize when an approach is unproductive and try a new approach.	**Apply mathematical tools to solve the problem.** W ■ Use strategies to solve problems (e.g., use number estimation — if one student reads 45 minutes [around 50] one night and if the same student reads 18 [around 20] minutes the next night, that is about 70 minutes). ■ Use appropriate tools to estimate solution (e.g., mental math or paper and pencil). ■ Recognize when an approach is unproductive and try a new approach.	**Apply mathematical tools to solve the problem.** W ■ Use strategies to solve problems (e.g., column addition, play money to determine costs, and subtraction to determine how much money is needed if they don't have enough). ■ Use appropriate tools to solve problems (e.g., paper and pencil, calculator, or physical models, play money). ■ Recognize when an approach is unproductive and try a new approach.

GLE	5	6	7	8	9/10
2.2.1	**Apply strategies, concepts, and procedures to devise a plan to solve the problem.** W ■ Gather and organize the necessary information or data from the problem (e.g., draw pictures, create a chart or table, or use models to organize information). ■ Determine what tools should be used to construct a solution (e.g., paper and pencil, pictures, physical models).	**Apply strategies, concepts, and procedures to devise a plan to solve the problem.** W ■ Organize relevant information from multiple sources to devise a plan (e.g., create a list of known and unknown information; create a table of values for length, width, and area of rectangles with perimeter of 100). ■ Select and apply appropriate mathematical tools for a situation (e.g., guess and check, creating tables of values [with or without technology], examine relationships between sides of a rectangle and area).	**Apply strategies, concepts, and procedures to devise a plan to solve the problem.** W ■ Organize relevant information from multiple sources (e.g., describe how to calculate percents, set limits on the number that Bonita could make). ■ Select and apply appropriate mathematical tools for a situation (e.g., guess and check, calculate Juan's percentage and create a table of values [with or without technology] for Bonita's percentage).	**Apply strategies, concepts, and procedures to devise a plan to solve the problem.** W ■ Organize relevant information from multiple sources. ■ Select and apply appropriate mathematical tools for a situation (e.g., plot steps per second vs. speed; check to see if model is linear; calculate successive differences or quotients to see if a pattern emerges; find an equation for a line that approximates the relationship or extend the pattern to approximate the speed at 5 steps per second).	**Apply strategies, concepts, and procedures to devise a plan to solve the problem.** W ■ Organize relevant information from multiple sources (e.g., create a list of known and unknown information; create a scatter plot of men's and women's times vs. time on the same coordinate axis to analyze the patterns). ■ Select and apply appropriate mathematical tools to devise a strategy in a situation (e.g., if the data, in either tabular or graphical form, suggest a linear relationship, plan to find a linear equation for each set of data; solve those equations simultaneously [or use technology to find the intersection of the two lines] to answer the question). If the data pattern suggests a non-linear model, plan to project what the pattern is and extend that pattern.
2.2.2	**Apply mathematical tools to solve the problem.** W ■ Use strategies to solve problems (e.g., draw pictures, use physical models). ■ Use appropriate tools to solve problems (e.g., paper and pencil, mental math, manipulatives). ■ Recognize when an approach is unproductive and try a new approach.	**Apply mathematical tools to solve the problem.** W ■ Implement the plan devised to solve the problem (e.g., in a table of values of lengths, widths, and areas find the one that shows the largest area; check smaller increments to see if this is the largest that works). ■ Identify when an approach is unproductive and modify or try a new approach (e.g., while guess and check may give some sense of a neighborhood of values, it is less efficient than a more organized method). ■ Check the solution to see if it works (e.g., if the solution gives a perimeter that is not 100, it makes no sense in the given problem).	**Apply mathematical tools to solve the problem.** W ■ Implement the plan devised to solve the problem or answer the question posed (e.g., in a table of values of percentages for Bonita's possible results and percentages, find the range of values that yield a percentage larger than Juan's; find the smallest of those and use that number). ■ Identify when an approach is unproductive and modify or try a new approach (e.g., if a result is larger than 25, return to see if the percentage computation is accurate and if it is computed correctly). ■ Check the solution to see if it works (e.g., if the solution is larger than 25, it makes no sense in the given problem).	**Apply mathematical tools to solve the problem.** W ■ Implement the plan devised to solve the problem or answer the question posed (e.g., in a table of values of lengths, widths, and areas find the one that shows the largest area; check smaller increments to see if this is the largest that works). ■ Identify when an approach is unproductive and modify or try a new approach (e.g., if an additive model didn't work, try a multiplicative model). ■ Check the solution to see if it works (e.g., if the solution for a speed of 19 feet per second is 5 steps per second, perhaps the assumption of linearity was incorrect).	**Apply mathematical tools to solve the problem.** W ■ Implement the plan devised to solve the problem (e.g., solve the set of simultaneous equations to arrive at a time where the two times are the same). ■ Use mathematics to solve the problem (e.g., use algebra to write equations for the two linear models, solve the system of equations using either symbols or technology). ■ Identify when an approach is unproductive and modify or try a new approach (e.g., if the result does not make sense in the context, return to the plan to see if something has gone wrong and adjust accordingly). ■ Check the solution to see if it works (e.g., the solution may be a partial year [i.e., 2003.6]; decide how to deal with this and also if the year is reasonable [i.e., 1925 does not make sense given the context]).

EALR 3: The student uses mathematical reasoning.

Component 3.1: Analyze information.

GLE	K	1	2	3	4
	Example: A classroom needs a playground ball for each student in the class. The class has fewer playground balls than are needed.	**Example:** A classroom is presenting a play and everyone has invited two guests. Enough chairs are needed to seat all the guests. There are some chairs in the classroom.	**Example:** A classroom is planning an all-day skating party on Thursday. Each student must pay for admission ($2), a box lunch ($3), and skate rental ($2). The teacher needs a total amount to reserve the rink.	**Example:** Miguel's reading class has set a goal to increase nightly reading to at least 30 minutes. He is taking a survey of his nine classmates to determine about how many minutes they read each night to see if they have met the goal. Miguel likes to read books by Matt Christopher.	**Example:** Jamal and his sister, Aleesha, want to buy a pet. Their mother said she will help by paying for the ongoing cost of food if they can save the money to buy the pet and all the needed equipment and food to get started. They have $18.73 saved already and most of that money is in quarters. They are reading pet store ads to see what the costs would be if they bought a mouse, a hamster, or a guinea pig.
3.1.1	**Understand how to compare information presented in familiar situations with teacher guidance.** ■ Restate understanding of the situation (e.g., each student requires a playground ball; there are not enough in the classroom).	**Understand how to compare information presented in familiar situations.** ■ Restate understanding of the situation (e.g., each guest attending the play will require a chair; there are not enough in the classroom).	**Understand how to compare information presented in familiar situations.** ■ Explain understanding of a situation, verbally or in writing (e.g., there are costs for admission, skates, lunch for the party; we need to know what it will cost for all of us so our teacher can reserve the rink). ■ Estimate how much money will be needed for all 25 students to attend.	**Analyze information presented in familiar situations.** W ■ Break down results from data to determine about how many minutes per night students are reading in order to estimate whether the class has met 30 minutes each night goal.	**Analyze information presented in familiar situations.** W ■ Break down the research information in order to explain or paraphrase it (e.g., each animal has costs related to cage, bedding, food which must be calculated in order to see if the kids have enough money to buy an animal).

GLE	5	6	7	8	9/10
	Example: Mrs. Allen's class won a pizza party sponsored by the PTA for best school attendance. There are 26 students in the class. Ten pizzas arrived but they were cut in three different ways. Three pizzas were cut in eighths, three were cut in fourths, and four were cut in halves. Mrs. Allen wouldn't let the students start eating until she was sure everyone received equal shares.				
3.1.1	**Analyze information in familiar situations.** W ■ Break down the research information in order to explain or paraphrase it (e.g., 26 students need to share ten pizzas equally. The pizzas are already sliced, but not evenly. Using eighths, determine how the pizza can be cut and shared equally).	**Analyze information from a variety of sources to interpret and compare information.** W ■ Identify claims based on statistical data and evaluate the validity of the claims. [1.4.5] ■ Read and interpret data from single line graphs and scatter plots and determine when the use of these graphs is appropriate. [1.4.5] ■ Use volume and capacity to describe and compare figures (e.g., fill containers with cubes to find which has a greater volume). [1.2.4]	**Analyze information from a variety of sources to interpret and compare information.** W ■ Explain and compare conclusions reached from data (e.g., from newspapers, web sites, opinion polls). [1.4.6] ■ Use graphs to describe trends, compare, and interpret relationships from data (e.g., from newspapers, web sites, opinion polls). [1.4.5]	**Analyze information from a variety of sources to interpret and compare information.** W ■ Predict the probability of outcomes of experiments and compare the prediction to empirical results. [1.4.2] ■ Predict an outcome given a linear relationship and a particular input (e.g., from a graph of profit projections, predict the profit in 2005). [1.5.1]	**Synthesize information from multiple sources in order to answer questions.** W ■ Use the properties of two-dimensional and three-dimensional figures to solve mathematical problems (e.g., find the width of a river based on similar triangles; given a set of parallel lines, a transversal, and an angle, find the other angles).

EALR 3: The student uses mathematical reasoning.

Component 3.2: Make predictions, inferences, conjectures, and draw conclusions.

GLE	K	1	2	3	4
	Example: A classroom needs a playground ball for each student in the class. The class has fewer playground balls than are needed.	**Example:** A classroom is presenting a play and everyone has invited two guests. Enough chairs are needed to seat all the guests. There are some chairs in the classroom.	**Example:** A classroom is planning an all-day skating party on Thursday. Each student must pay for admission ($2), a box lunch ($3), and skate rental ($2). The teacher needs a total amount to reserve the rink.	**Example:** Miguel's reading class has set a goal to increase nightly reading to at least 30 minutes. He is taking a survey of his nine classmates to determine about how many minutes they read each night to see if they have met the goal. Miguel likes to read books by Matt Christopher.	**Example:** Jamal and his sister, Aleesha, want to buy a pet. Their mother said she will help by paying for the ongoing cost of food if they can save the money to buy the pet and all the needed equipment and food to get started. They have $18.73 saved already and most of that money is in quarters. They are reading pet store ads to see what the costs would be if they bought a mouse, a hamster, or a guinea pig.
3.2.1	**Understand how to make a reasonable prediction based on the information given in a familiar situation.** ■ Predict a numerical solution for a problem (e.g., guess how many more playground balls are needed).	**Understand how to make a reasonable prediction based on prior knowledge and the information given in a familiar situation.** ■ Predict a numerical solution for a problem (e.g., predict how many more chairs will be needed). ■ Use known information to make a reasonable prediction (e.g., if two numbers are each less than 10, the sum will be less than 20). ■ Make an inference based on information provided (e.g., the boys in class did a better job convincing their guests to attend because there are more guests coming for the boys than the girls).	**Understand how to make a reasonable prediction based on prior knowledge and the information given in a familiar situation.** ■ Predict a numerical solution for a problem (e.g., predict how much it will cost for the class to attend the skating party). ■ Use known information to make a reasonable prediction (e.g., if most students in one class like red apples, then most students in another class will like red apples). ■ Make an inference based on information provided (e.g., when you skate at the rink with a big group it costs less for each person than when you go with a friend).	**Apply prediction and inference skills.** W ■ Make a reasonable prediction based on prior knowledge and investigation of situation (e.g., after collecting survey data and before estimation, predict whether the class will meet its goal). ■ Defend prediction with evidence from the situation. ■ Make inferences (conjectures) using information from the situation to support the inference (e.g., the class probably did not make the reading goal because the community softball league has started up and most kids are involved in the evenings).	**Apply prediction and inference skills.** W ■ Make a reasonable prediction based on prior knowledge and investigation of situation (e.g., after reading the pet store ads, predict whether the kids will be able to buy a pet). ■ Defend prediction with evidence from the situation. ■ Make inferences (conjectures) using information from the situation or data to support the inference (e.g., guinea pig equipment/food is more expensive because the animal is larger and requires a bigger cage and pellets).

GLE	5	6	7	8	9/10
	Example: Mrs. Allen's class won a pizza party sponsored by the PTA for best school attendance. There are 26 students in the class. Ten pizzas arrived but they were cut in three different ways. Three pizzas were cut in eighths, three were cut in fourths, and four were cut in halves. Mrs. Allen wouldn't let the students start eating until she was sure everyone received equal shares.				
3.2.1	**Apply prediction and inference skills.** W ■ Make a reasonable prediction based on prior knowledge and investigation of situation (e.g., using mental math, predict how many pieces each student will receive). ■ Defend prediction with evidence from the situation. ■ Make inferences (conjectures) using information from the situation or data to support the inference (e.g., all the pizzas were the same size when whole).	**Apply prediction and inference skills to make or evaluate conjectures.** W ■ Identify claims based on statistical data and evaluate the validity of the claims. [1.4.5] ■ Predict a future element in a relation (e.g., find the fifteenth term in a pattern). [1.5.1]	**Apply prediction and inference skills to make or evaluate conjectures.** W ■ Predict the probability of future events based on empirical data. [1.4.2] ■ Predict the probability of outcomes of experiments and test the predictions. [1.4.2]	**Apply prediction and inference skills to make or evaluate conjectures.** W ■ Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken (e.g., age groups, regions of the U.S., genders, racial/ethnic distribution). [1.4.6]	**Apply skill of conjecturing and analyze conjectures by formulating a proof or constructing a counter example.** W ■ Make and test conjectures about two-dimensional and three-dimensional figures and their individual attributes and relationships using physical, symbolic, and technological models (e.g., diagonal of a rectangle or prism is the longest interior segment; what figures make up cross-sections of a given three-dimensional shape). [1.3.1]

EALR 3: The student uses mathematical reasoning.

Component 3.2: Make predictions, inferences, conjectures, and draw conclusions.

GLE	K	1	2	3	4
3.2.2		**Understand how to draw conclusions based on prior knowledge and the information given in a familiar situation.** ■ Draw conclusions from displays using comparative language (e.g., more students have two guests coming, or fewer students have only one guest coming) and provide examples from displays to support conclusions.	**Understand how to draw conclusions based on prior knowledge and the information given in a familiar situation.** ■ Draw conclusions from displays using comparative language (e.g., greater than, less than). ■ Provide data to justify conclusions. ■ Provide examples from displays to support conclusions.	**Apply the skills of drawing conclusions and support the conclusions using evidence.** W ■ Draw conclusions from displays, texts, or oral discussions and justify those conclusions with logical reasoning or other evidence.	**Apply the skills of drawing conclusions and support the conclusions using evidence.** W ■ Draw conclusions from displays, texts, or oral discussions and justify those conclusions with logical reasoning or other evidence.
3.2.3		**Analyze procedures used to solve problems in familiar situations with teacher guidance.** ■ Justify the importance of counting in a situation rather than making a guess at a number of items for a specific purpose (e.g., counting the number of chairs needed for the play rather than guessing).	**Analyze procedures used to solve problems in familiar situations.** ■ Justify the use of a chart or table to collect and organize information used to solve a problem (e.g., the two- or four-column chart helped to keep track of the information). ■ Justify the use of one mathematical tool over another (e.g., is a calculator or 100's chart a better tool in this situation?).	**Analyze procedures used to solve problems in familiar situations.** W ■ Describe and compare estimation strategies used (e.g., front end estimation vs. using compatible numbers). [1.1.8]	**Analyze procedures used to solve problems in familiar situations.** W ■ Describe and compare data organization methods (e.g., charts used for organizing costs for each animal). [1.4.3]

Component 3.2: Make predictions, inferences, conjectures, and draw conclusions.

GLE	5	6	7	8	9/10
3.2.2	**Apply the skills of drawing conclusions and support the conclusions using evidence.** W ■ Draw conclusions from displays, texts, or oral discussions and justify those conclusions with logical reasoning or other evidence.	**Apply the skills of drawing conclusions and support those conclusions using evidence.** W ■ Draw conclusions from displays, texts, or oral discussions and justify those conclusions with logical reasoning or other evidence (e.g., read a newspaper article or ad; draw a conclusion and support that conclusion with evidence from the article or elsewhere).	**Apply the skills of drawing conclusions and support those conclusions using evidence.** W ■ Draw conclusions from displays, texts, or oral discussions and justify those conclusions with logical reasoning or other evidence (e.g., read a newspaper article that includes data, draw a conclusion, and support that conclusion with evidence from the article or elsewhere).	**Apply the skills of drawing conclusions and support those conclusions using evidence.** W ■ Draw conclusions from displays, texts, or oral discussions and justify those conclusions with logical reasoning or other evidence (e.g., read an editorial or ad, draw a conclusion and support that conclusion with evidence in the article or elsewhere).	**Analyze information to draw conclusions and support them using inductive and deductive reasoning.** W ■ Compare and describe the volume of cylinders, cones, and prisms when an attribute is changed (e.g., the area of the base, the height of solid). [1.2.4] ■ Draw a plane shape of a given set of characteristics and justify the answer. [1.3.2] ■ Identify trends in a set of data in order to make a prediction based on the information. [1.4.6] ■ Use statistics to support different points of view. [1.4.6]
3.2.3	**Analyze procedures used to solve problems in familiar situations.** W ■ Describe and compare strategies and tools used (e.g., drawing pizzas, fraction wheels or strips, paper and pencil calculations).	**Analyze procedures and results in various situations.** W ■ Represent and interpret all possible outcomes of experiments (e.g., an organized list, a table, a tree diagram, or a sample space). [1.4.2]	**Analyze procedures and results in various situations.** W ■ Describe how additional data added to data sets may affect the computations of measures of central tendency in various situations. [1.4.4]	**Analyze procedures and results in various situations.** W ■ Critique conclusions drawn from a set of data and support with evidence (e.g., from magazines, newspapers, web sites, opinion polls). [1.4.6]	**Analyze procedures to determine appropriateness of claims and arguments.** W ■ Examine claims and supporting arguments based on data and make needed revisions. [1.4.6]

EALR 3: The student uses mathematical reasoning.

Component 3.3: Verify results.

GLE	K	1	2	3	4
	Example: A classroom needs a playground ball for each student in the class. The class has fewer playground balls than are needed.	**Example:** A classroom is presenting a play and everyone has invited two guests. Enough chairs are needed to seat all the guests. There are some chairs in the classroom.	**Example:** A classroom is planning an all-day skating party on Thursday. Each student must pay for admission ($2), a box lunch ($3), and skate rental ($2). The teacher needs a total amount to reserve the rink.	**Example:** Miguel's reading class has set a goal to increase nightly reading to at least 30 minutes. He is taking a survey of his nine classmates to determine about how many minutes they read each night to see if they have met the goal. Miguel likes to read books by Matt Christopher.	**Example:** Jamal and his sister, Aleesha, want to buy a pet. Their mother said she will help by paying for the ongoing cost of food if they can save the money to buy the pet and all the needed equipment and food to get started. They have $18.73 saved already and most of that money is in quarters. They are reading pet store ads to see what the costs would be if they bought a mouse, a hamster, or a guinea pig.
3.3.1	**Understand how to justify results using evidence.** ■ Use tools (e.g., tally marks, physical models, words) to check for reasonableness of an answer (e.g., line up students, pass out the playground balls to students to see how many students do not receive one). ■ Check reasonableness of an estimation by acting it out, using pictures, or physical models.	**Understand how to justify results using evidence.** ■ Check reasonableness of results by using pictures, physical models, or acting it out (e.g., students raise one hand for one guest attending and two hands if two guests are attending).	**Understand how to justify results using evidence.** ■ Check for reasonableness of results by using a calculator for repeated addition (e.g., to determine the total cost of the skating party).	**Understand how to justify results using evidence.** W ■ Check for reasonableness of results by using a different strategy or tool to solve the problem (e.g., use front end estimation to determine about how many minutes students were reading each night). ■ Justify whether estimation is appropriate for the situation.	**Understand how to justify results using evidence.** W ■ Check for reasonableness of results by using a different strategy or tool to solve the problem (e.g., use front end estimation to determine about how much each animal will cost). ■ Provide examples to support results.
3.3.2		**Understand how to validate thinking about numerical, measurement, geometric, or statistical ideas by using models, known facts, patterns, or relationships.** ■ Explain why a strategy or tool was used in solving a problem (e.g., why a two-column chart was helpful to gather the information needed about the number of guests attending the play).	**Understand how to validate thinking about numerical, measurement, geometric, or statistical ideas by using models, known facts, patterns, or relationships.** ■ Explain why a strategy or tool was used in solving a problem (e.g., why a seating chart was helpful to determine total cost of skating).	**Understand how to validate thinking about numerical, measurement, geometric, or statistical ideas by using models, known facts, patterns, or relationships.** W ■ Explain how comparisons can be used to draw a conclusion (e.g., the class won't have met the reading goal because fewer students read less than more this week and didn't make the goal last week).	**Understand how to validate thinking about numerical, measurement, geometric, or statistical ideas by using models, known facts, patterns, or relationships.** W ■ Explain the meaning of a decimal using physical models. [1.1.5] ■ Explain what the relative position of numbers on a positive number line means (e.g., to the right means greater than). [1.3.3]

GLE	5	6	7	8	9/10
	Example: Mrs. Allen's class won a pizza party sponsored by the PTA for best school attendance. There are 26 students in the class. Ten pizzas arrived but they were cut in three different ways. Three pizzas were cut in eighths, three were cut in fourths, and four were cut in halves. Mrs. Allen wouldn't let the students start eating until she was sure everyone received equal shares.				
3.3.1	**Understand how to justify results using evidence.** W ■ Check for reasonableness of results by using a different strategy or tool to solve the problem (e.g., compare the results from students who used physical models vs. those who used computation). ■ Provide examples to support results.	**Analyze procedures and information used to justify results using evidence.** W ■ Find and compare rectangular prisms that have a given volume (e.g., if two rectangular prisms have the same volume and one has twice the height of the other, determine how the areas of their bases compare). [1.2.5] ■ Apply estimation strategies prior to computation of whole numbers, decimals, and fractions to determine reasonableness of answers. [1.1.8] ■ Identify different ways of selecting a sample (e.g., convenience sampling, response to a survey, random sampling) and which method makes a sample more representative for a population. [1.4.3]	**Analyze procedures and information used to justify results using evidence.** W ■ Justify the reasonableness of an estimate. [1.2.6] ■ Apply a process that can be used to find a reasonable estimate of circle measurements (e.g., wrap a string around the circle). [1.2.6] ■ Apply estimation strategies prior to computing addition and subtraction of integers and operations on non-negative rational numbers to determine reasonableness of answers. [1.1.8]	**Analyze procedures and information used to justify results using evidence.** W ■ Use estimation to predict or to verify the reasonableness of calculated results. [1.1.8]	**Analyze results using inductive and deductive reasoning.** W ■ Compare and contrast similar two-dimensional figures and shapes using properties of two-dimensional figures and shapes. [1.3.2] ■ Find a reasonable estimate for the volume of prisms, pyramids, cylinders, and cones. [1.2.6]
3.3.2	**Understand how to validate thinking about numerical, measurement, geometric, or statistical ideas by using models, known facts, patterns, or relationships.** W ■ Explain how the value of a fraction changes in relationship to the size of the whole (e.g., half a pizza vs. half a cookie). [1.1.1] ■ Create three-dimensional shapes from two-dimensional figures (e.g., cylinder from two circles and a rectangle) and explain the relationship. [1.3.2]	**Analyze thinking and mathematical ideas using models, known facts, patterns, relationships, or counter examples.** W ■ Identify claims based on statistical data and evaluate the validity of the claims. [1.4.5]	**Analyze thinking and mathematical ideas using models, known facts, patterns, relationships, or counter examples.** W ■ Explain how different representations of the same set of data can support different points of view. [1.4.6]	**Analyze thinking and mathematical ideas using models, known facts, patterns, relationships, or counter examples.** W ■ Explain why a given rational number is greater than or less than another rational number. [1.1.2]	**Analyze thinking and mathematical ideas using models, known facts, patterns, relationships, counter examples, or proportional reasoning.** W ■ Examine a set of data, research other sources to see if the data is consistent, find articles written to see if the data makes sense, to develop a logical point of view and to support that view. [1.4.6]

EALR 4: The student communicates knowledge and understanding in both everyday and mathematical language.

Component 4.1: Gather information.

GLE	K	1	2	3	4
4.1.1		**Understand how to develop and follow a simple plan for collecting information for a given purpose.** ■ Determine what information is needed and how to collect it for a given purpose (e.g., to help explain something, to find out if something is needed) and who the information is for (e.g., for the classroom, for the adults at home, for the librarian). ■ Develop and follow a plan to gather data about an event (e.g., how many students will attend the Saturday Movie Afternoon at school?).	**Understand how to develop and follow a simple plan for collecting information for a given purpose.** ■ Determine what information is needed and how to collect it for a given purpose (e.g., to help explain something, to find out if something is needed) and who the information is for (e.g., for the classroom, for the adults at home, for the cafeteria, for the principal). ■ Develop and follow a plan to gather information about supplies needed for a project (e.g., how many pieces of paper will be needed to create a pattern design for each of the kindergarten windows?).	**Understand how to follow a plan for collecting information for a given purpose.** W ■ Determine how to collect information for a specific purpose or audience (e.g., to convince a parent or other adult, to demonstrate a need for change, to provide information). ■ Develop and follow a plan based on the kind of information needed, the purpose, and the audience (e.g., survey, gather data from a chart or graph, read in a text to gather information).	**Understand how to develop and follow a plan for collecting information for a given purpose.** W ■ Determine how to collect information for a specific purpose or audience (e.g., to convince a parent or other adult, to demonstrate a need for change, to provide information). ■ Develop and follow a plan based on the kind of information needed, the purpose, and the audience (e.g., survey, gather data from a chart or graph, read in a text to gather information).
4.1.2		**Understand how to extract information for a given purpose from one or two different sources.** ■ Follow simple written directions for creating an art project using a model (e.g., requiring cutting and folding geometric shapes). ■ Generate questions that could be answered using informational text (e.g., TV ads, books, menus, cereal boxes).	**Understand how to extract information for a given purpose from one or two different sources.** ■ Decide what information would be important to learn about the students in the second grade after reading an informational text (e.g., health article) in class (e.g., how many students eat a nutritious breakfast). Determine what questions to ask in a survey. Graph the results.	**Understand how to extract information for a given purpose from one or two different sources using reading, listening, and observation.** W ■ Read and report on data from tables, charts, and bar graphs. [1.4.5] ■ Read directions for movement on a positive number line, identify the point of final destination using real-world examples (e.g., travel back and forth on a street, temperature variations during the day). [1.3.3]	**Understand how to extract information for a given purpose from one or two different sources using reading, listening, and observation.** W ■ Listen and observe to simulate translations and reflections using objects (e.g., pattern blocks, geo blocks). [1.3.4] ■ Read and follow directions using a coordinate grid (e.g., on a city street map). [1.3.3]

GLE	5	6	7	8	9/10
4.1.1	**Understand how to develop and follow a plan for collecting information for a given purpose.** W ■ Determine how to collect information for a specific purpose or audience (e.g., to convince a parent or other adult, to demonstrate a need for change, to provide information). ■ Develop and follow a plan based on the kind of information needed, the purpose, and the audience (e.g., survey, gather data from a chart or graph, read in a text to gather information). ■ Ask the same question using different data collection methods that result in other points of view being supported. [1.4.3] ■ Explain how different data collection methods affect the nature of the data set with a given question (e.g., phone survey, person-to-person survey, nternet search). [1.4.3]	**Apply a planning process to collect information for a given purpose.** W ■ Use mean, median, and mode to explain familiar situations (e.g., the heights of students in the class; the hair color of students in the class). [1.4.4] ■ Decide on information needed to create a report on a mathematical topic (e.g., compare the predicted rainfall in a given period with the actual rainfall).	**Apply a planning process to collect information for a given purpose.** W ■ Formulate a question and collect data from a population considering how the questions, collection method, and sample population affect the results. [1.4.3]	**Apply a planning process to collect information for a given purpose.** W ■ Describe a procedure for selecting an unbiased sample. [1.4.3]	**Understand how to develop or apply an efficient system for collecting mathematical information for a given purpose.** W ■ Collect data efficiently on the outcomes of first events and later events to determine and justify how the first event affects the probability of later events (e.g., drawing cards from a deck with or without replacement). [1.4.1]
4.1.2	**Understand how to extract information for a given purpose from one or two different sources using reading, listening, and observation.** W ■ After reading a text, generate questions and develop a survey (e.g., to determine how many students agree or disagree with the author). ■ Identify and use data from text passages, histograms, stem-and-leaf plots, and circle graphs. [1.4.5]	**Understand how to extract information from multiple sources using reading, listening, and observation.** W ■ Use mean, median, and mode to explain situations (e.g., the heights of students in the class; hair color of students in the class; favorite movie of students in the class; most watched movie in a specific time frame). [1.1.4]	**Understand how to extract information from multiple sources using reading, listening, and observation.** W ■ Create a table or graph given a description of, or an equation for, a situation involving a linear or non-linear relationship. [1.5.4]	**Synthesize information from multiple sources using reading, listening, and observation.** W ■ Compare the results of a survey given two different sample groups. [1.4.3] ■ Model the relationship with a table or graph given a description of, or an equation for, a situation involving an inequality or linear relationship. [1.5.4]	**Synthesize mathematical information for a given purpose from multiple, self-selected sources.** W ■ State possible factors that may influence a trend but not be reflected in the data (e.g., population growth of deer vs. availability of natural resources or hunting permits). [1.4.6]

EALR 4: The student communicates knowledge and understanding in both everyday and mathematical language.

Component 4.2: Organize, represent, and share information.

GLE	K	1	2	3	4
4.2.1	**Understand how to organize information to communicate to a given audience with teacher guidance.** ■ Use a two-column chart to organize data (e.g., one column for student names and tally marks in the other to represent which students are assigned a ball) for the classroom with teacher guidance. ■ Use physical objects or pictures to build bar graphs to answer a question generated by the class (e.g., how many of each kind of pet do we own?).	**Understand how to organize information to communicate to a given audience with teacher guidance.** ■ Organize and display data on a chart to communicate a solution for the given audience (e.g., use a two- or three-column chart to display the number of guests per student attending a class play and, if there is a chair for each guest, inform the custodian as to how many more chairs are needed). ■ Display results of data collection by making student-invented and conventional displays (e.g., hair color, eye color, teeth missing).	**Understand how to organize information to communicate to a given audience.** ■ Organize and display data on a chart to communicate a solution to a specific audience (e.g., use a chart to display individual costs and total cost for the skating party for parents and PTA). ■ Construct a bar graph with a title, key, and single unit increment to display survey results (e.g., the number of brothers and sisters of students in the class).	**Understand how to organize information for a given purpose.** W ■ Create a display to represent information from survey results (e.g., the approximate number of minutes read and whether or not the goal was met). ■ Create bar graphs including labels for title, both axes, scale units (e.g., 2's, 5's, 10's), and key if needed. [1.4.2] ■ Create and solve a problem situation where mode is meaningful for a set of data. [1.4.4] ■ Display information to be shared.	**Understand how to organize information for a given purpose.** W ■ Organize information on a chart and create a summary of the results to inform a specific audience (e.g., chart all related costs for the purchase of each pet; write a summary explaining the results and the kids' possible decisions based on the results). ■ Construct assorted line and pictographs that include labels, a scale that is not one, and a key. [1.4.5] ■ Create a chart or display to represent equivalent fractions. [1.1.2]
4.2.2	**Understand how to communicate or represent ideas or information using mathematical language or notation.** ■ Explain or represent ideas using mathematical language from: – Number sense (e.g., numbers 1 to 10) [1.1.1]; – Measurement (e.g., compare objects to describe relative size) [1.2.1]; – Geometric sense (e.g., name objects based on their characteristics — I have four equal sides; what am I?) [1.3.1]; – Algebraic sense (e.g., create a pattern such as AB). [1.5.1]	**Understand how to communicate or represent ideas or information using mathematical language or notation.** ■ Explain or represent ideas using mathematical language from: – Number sense (e.g., numbers to at least 100) [1.1.1]; – Measurement (e.g., order three or more objects according to an attribute and identify the chosen attribute) [1.2.1]; – Geometric sense (e.g., name and describe two-dimensional figures based on their characteristics) [1.3.1]; – Statistics (e.g., construct bar graphs with physical materials) [1.4.3]; – Algebraic sense (e.g., explain the meaning of equality). [1.5.3]	**Understand how to communicate or represent ideas or information using mathematical language or notation.** ■ Explain or represent ideas using mathematical language from: – Number sense (e.g., numbers to at least 1000) [1.1.1]; – Measurement (e.g., identify attributes of an object that are measurable — time, length, distance around, capacity, area or weight of objects) [1.2.1]; – Geometric sense (e.g., describe characteristics of two-dimensional geometric figures, various polygons) [1.3.1]; – Statistics (e.g., construct bar graph using a single increment scale) [1.4.3]; – Algebraic sense (e.g., explain and use the symbols < and > to express relationships). [1.5.3]	**Understand how to communicate or represent ideas using mathematical language or notation.** W ■ Translate from one representation of a whole number to another in standard, expanded, and word forms. [1.1.1] ■ Name attributes of an object that can be measured. [1.2.4] ■ Identify, describe, and compare congruent two-dimensional geometric shapes. [1.3.1] ■ Make a survey and collect data (e.g., use tally marks, make a table). [1.4.3] ■ Identify and use appropriate symbols and notation in reading and writing simple expressions and equations involving addition and subtraction. [1.5.4]	**Understand how to communicate or represent ideas using mathematical language or notation.** W ■ Symbolically represent parts of a whole or parts of a set with common denominators. [1.1.1] ■ Use measurements of area to describe and compare objects. [1.2.1] ■ Describe a location in the first quadrant on a coordinate grid in terms of horizontal and vertical position (e.g., to the right and up, longitude and latitude). [1.3.3] ■ Describe a trend from a given line plot. [1.4.5] ■ Describe the rule for a pattern with a single arithmetic operation in the rule. [1.5.2]

GLE	5	6	7	8	9/10
4.2.1	**Understand how to organize information for a given purpose.** W ■ Determine the best method for organizing and representing information for a specific purpose (e.g., a physical model or a calculation to inform the teacher how many pieces of pizza each student should receive). ■ Represent and interpret all possible outcomes of experiments (e.g., an organized list, a table, a tree diagram, or a sample space). [1.4.2] ■ Construct assorted graphs including histograms, pictographs, and stem-and-leaf plots that include labels, appropriate scale, and key. [1.4.5]	**Apply organizational skills for a given purpose.** W ■ Show the order of the set of integers on a number line with both positive and negative numbers (e.g., organize the given birth years of the following Arabic kings on a number line). [1.3.3]	**Apply organizational skills for a given purpose.** W ■ Identify, determine, interpret, or express probabilities in the form of a fraction, decimal, or percent. [1.4.2]	**Apply organizational skills for a given purpose.** W ■ Design and conduct a simulation, with and without technology, to determine the probability of an event occurring. [1.4.2]	**Analyze mathematical information to organize, clarify, and refine an argument.** W ■ Develop an argument to support a given point of view and set of statistics. [1.4.6]
4.2.2	**Understand how to communicate or represent ideas using mathematical language or notation.** W ■ Explain the value of a given digit in a decimal to at least the thousandths place. [1.1.1] ■ Describe a procedure for measuring an angle. ■ Describe relationships between angle measures (e.g., two 30° angles have the same total measure as one 60° angle). [1.2.2] ■ Draw and label a design that includes a given set of attributes. [1.3.2] ■ Explain how to find the mean of a set of data and explain the significance of the mean. [1.4.4] ■ Given an expression or equation, identify or write a situation that represents it. [1.5.3]	**Apply communication skills to clearly and effectively express or present ideas and situations using mathematical language or notation.** W ■ Articulate various strategies used during estimation involving fractions and decimals. [1.1.8] ■ Clearly explain, describe, or represent mathematical information in a pictorial, tabular, graphical, two- or three-dimensional drawing, or other form as appropriate for the mathematical information (e.g., time, distance, categories), audience, and/or purpose, such as to perform or persuade, with notation and labels as needed. ■ Use an appropriate representation to display data (e.g., table, graphs) given a particular situation and audience. [1.4.5]	**Apply communication skills to clearly and effectively express or present ideas and situations using mathematical language or notation.** W ■ Identify data that may represent sampling errors and explain why the sample (and the display) might be biased. [1.4.4] ■ Explain when estimation might be used rather than computation. [1.1.8] ■ Clearly explain, describe, or represent mathematical information in a pictorial, tabular, graphical, two- or three-dimensional drawing, or other form as appropriate for the mathematical information (e.g., time, distance, categories), audience, and/or purpose such as to perform or persuade with notation and labels as needed.	**Apply communication skills to clearly and effectively express or present ideas and situations using mathematical language or notation.** W ■ Articulate various strategies used during estimation involving integers. [1.1.8] ■ Clearly explain, describe, or represent mathematical information in a pictorial, tabular, graphical, two- or three-dimensional drawing, or other form as appropriate for the mathematical information (e.g., time, distance, categories), audience, and/or purpose, such as to perform or persuade, with notation and labels as needed. ■ Explain situations involving real numbers where estimates are sufficient and others for which exact value is required. [1.1.8]	**Understand how to express ideas and situations using mathematical language and notation.** W ■ Explain how division of measurements produces a derived unit of measurement (e.g., miles traveled divided by hours traveled yields the derived unit [miles per hour]). [1.2.2] ■ Describe the location of points that satisfy given conditions (e.g., the set of points equidistant from a given point; a point equidistant from a given set of points). [1.3.3] ■ Describe and compare the impact that a change in one or more dimensions has on objects (e.g., doubling the edge of a cube affects the surface area). [1.2.1] ■ Explain the relationship between theoretical probability and empirical frequency of dependent events using simulations with and without technology. [1.4.2]

EALR 5: The student understands how mathematical ideas connect within mathematics, to other subject areas, and to real-life situations.

Component 5.1: Relate concepts and procedures within mathematics.

GLE	K	1	2	3	4
5.1.1	**Understand how to use concepts and procedures from any two of the content components from EALR 1 in a given problem or situation.** ■ Organize data collections (e.g., bar graph, sorted groups) and compare data using comparative language. [1.1.2, 1.4.3] ■ Sort objects based on chosen attribute and create a simple AB pattern using the sorted objects. [1.3.2, 1.5.1]	**Understand how to use concepts and procedures from any two of the content components from EALR 1 in a given problem or situation.** ■ Interpret results and draw conclusions from student-made displays using comparative language (e.g., more, fewer). [1.4.4, 3.2.2] ■ Measure objects using non-standard tools and place resulting numbers in order from shortest (smallest) to longest (largest). [1.2.3, 1.1.2]	**Understand how to use concepts and procedures from any two of the content components from EALR 1 in a given problem or situation.** ■ Conduct a survey for a predetermined question, collect data, and use addition and subtraction procedures to compute the results of the survey. [1.4.4, 1.1.6] ■ Interpret a bar graph for comparative information (e.g., how many more than, less than) and draw conclusions about the data. [1.4.5, 3.2.2]	**Understand how to use concepts and procedures from any two of the content components in a given problem or situation.** W ■ Conduct a survey for a question, collect data, and use three-digit addition and subtraction to compute the results of the survey. [1.1.6, 1.4.4] ■ Explain and use a method for making change with coins. [1.1.1, 1.2.4]	**Understand how to use concepts and procedures from any two of the content components in a given problem or situation.** W ■ Conduct a survey for a question; collect data, and use multiplication and/or division to compute the results of the survey. [1.1.6, 1.4.4] ■ Identify, describe, and compare attributes of congruent shapes in multiple orientations. [1.3.2]
5.1.2	**Understand how to recognize and create equivalent mathematical models and representations in familiar situations.** ■ Identify different representations of a number to 20 (e.g., numerals, pictures, physical models). [1.1.1] ■ Express stories involving addition (e.g., join) with models, pictures, and symbols. [1.1.5]	**Understand how to recognize and create equivalent mathematical models and representations in familiar situations.** ■ Identify different representations of a number to at least 100 (e.g., numerals, pictures, physical models). [1.1.1] ■ Express stories involving subtraction (e.g., separate) with models, pictures, and symbols. [1.1.5]	**Understand how to recognize and create equivalent mathematical models and representations in familiar situations.** ■ Represent addition and subtraction situations with physical models, diagrams, and acting out problems. [1.1.5] ■ Identify different representations of a pattern (e.g., snap-clap-stomp translates to ABC). [1.5.1]	**Understand how to recognize equivalent mathematical models and representations in familiar situations.** W ■ Translate from one representation of a whole number to another in standard, expanded, and word forms. [1.1.1] ■ Compare strategies to solve problems involving multiplication and division (e.g., alternative algorithms, use of properties of multiplication). [1.1.5] ■ Use the inverse relationship between multiplication and division using physical diagrams, words, and symbols (e.g., arrays, fact families). [1.1.5]	**Understand how to recognize equivalent mathematical models and representations in familiar situations.** W ■ Demonstrate and explain equivalent relationships between decimals and fractions (e.g., $.50 is equal to ½ a dollar and 50/100 dollar) using models. [1.1.2] ■ Interpret remainders of a division problem in a given situation (e.g., remainder 3 or 3/5). [1.1.6] ■ Represent addition and subtraction of decimals through hundredths using models (e.g., base ten blocks, fraction circles with decimal ring, money). [1.1.6]

Component 5.1: Relate concepts and procedures within mathematics.

GLE	5	6	7	8	9/10
5.1.1	**Understand how to use concepts and procedures from any two of the content components in a given problem or situation.** W ■ Explain why angle measure does not change when the size of the circle or length of the sides of the angle change. [1.2.3] ■ Interpret skew, clusters, and gaps in given one-variable data displays. [1.4.5] ■ Translate a situation involving one arithmetic operation into algebraic form using equations, tables, and graphs. ■ Judge the appropriateness of inferences made from a set of data and support the judgment. [1.4.6]	**Apply concepts and procedures from a variety of mathematical areas in a given problem or situation.** W ■ Translate a situation involving multiple arithmetic operations into algebraic form using equation, table, and graphs. [1.5.4] ■ Given a set of data, compare various representations (e.g., table, graph, rule) for a given situation. [1.4.5]	**Apply concepts and procedures from a variety of mathematical areas in a given problem or situation.** W ■ Write the rational number when given a model (e.g., number line, area model, situation, diagram, picture). [1.1.1] ■ Given a set of data, compare various representations (e.g., box-and-whisker, bar, circle graph) for a given situation. [1.4.5]	**Apply concepts and procedures from a variety of mathematical areas in a given problem or situation.** W ■ Solve problems involving ratio and proportion (e.g., similar figures, scale drawings, rates, find unit pricing, increase or decrease a recipe, find the portions for a group converting between different units of measure, or finding medicinal dosages). [1.1.4] ■ Find the area of a circle given the coordinates of the center and a point on the circle. [1.3.3]	**Apply multiple mathematical concepts and procedures in a given problem or situation.** W ■ Estimate derived units of measure (e.g., miles per hour, people/year, grams/cubic centimeters). [1.2.6] ■ Determine the final coordinates of a point after a series of transformations. [1.3.4]
5.1.2	**Understand how to recognize equivalent mathematical models and representations in familiar situations.** W ■ Use factors and multiples to rename equivalent fractions. [1.1.1] ■ Determine equivalence among fractions. [1.1.2] ■ Graphically represent the same data in two different ways.	**Apply different mathematical models and representations to the same situation.** W ■ Represent equivalent ratios or given percentages using objects, pictures, and symbols. [1.1.4] ■ Match a graph with a data set. [1.5.4]	**Apply different mathematical models and representations to the same situation.** W ■ Explain how different representations of the same set of data can support different points of view. [1.4.6] ■ Match a situation with a data set or graph. [1.5.4]	**Apply different mathematical models and representations to the same situation.** W ■ Create a problem situation to match a given rational number equation. [1.1.5] ■ Match a situation with a data set or graph. [1.5.4]	**Understand how to use different mathematical models and representations in the same situation.** W ■ Identify, interpret, and use the meaning of slope of a line as a rate of change using concrete, symbolic, and technological models. [1.2.2] ■ Construct one-dimensional, two-dimensional, and three-dimensional geometric figures using a variety of tools and technologies (e.g., angle bisectors, perpendicular bisectors, triangles given specific characteristics). [1.3.2] ■ Find the equation of a line in a variety of ways (e.g., from a table, graph, slope-intercept, point-slope, two points). [1.5.1] ■ Find the solution to a system of linear equations using tables, graphs and symbols. [1.5.6]

EALR 5: The student understands how mathematical ideas connect within mathematics, to other subject areas, and to real-life situations.

Component 5.2: Relate mathematical concepts procedures to other disciplines.

GLE	K	1	2	3	4
5.2.1	**Apply and analyze the use of mathematical patterns and ideas in familiar situations in other disciplines.** ■ Describe how math is used in science when a number of objects are needed for an experiment or measurement is used to illustrate change. ■ Identify patterns in a piece of artwork.	**Apply and analyze the use of mathematical patterns and ideas in familiar situations in other disciplines.** ■ Use the characteristics of two-dimensional shapes in art projects and recognize the use of geometric shapes in artwork. ■ Use a clock to determine when it is time for recess or lunch time. ■ Explain how math is used whenever we use money for a purchase.	**Apply and analyze the use of mathematical patterns and ideas in familiar situations in other disciplines.** ■ Collect and display data based on a science experiment (e.g., plant growth, magnetism). ■ Identify patterns used in the design of common objects (e.g., skateboards, clothing). ■ Describe how estimation can be used to know about how much something costs.	**Apply mathematical patterns and ideas in familiar situations in other disciplines.** ■ Given an object, identify geometric attributes that can be measured. ■ Interpret graphs for comparative information. [1.4.3] ■ Pose questions and gather data about self and surroundings. [1.4.3] ■ Make inferences based on data or determine if the data can support inferences made. [1.4.5]	**Apply mathematical patterns and ideas in familiar situations in other disciplines.** ■ Read and interpret data from line plots and pictographs. [1.4.5] ■ Make a plan to answer a question including how to record and organize data. [1.4.3] ■ Use estimation strategies appropriately when the exact answer is not necessary. [1.1.7]
5.2.2		**Know the contributions of individuals and cultures to the development of mathematics.** ■ Recognize the contributions of women, men, and people from different cultures (e.g., look at symbols used for numbering in the Mayan culture).	**Know the contributions of individuals and cultures to the development of mathematics.** ■ Recognize the contributions of women, men, and people from different cultures (e.g., examine design and patterns on tapestry from various African cultures).	**Know the contributions of individuals and cultures to the development of mathematics.** ■ Recognize the contributions to the development of mathematics by women, men, and various cultures (e.g., complete a mathematically based project that researches the history of 0).	**Know the contributions of individuals and cultures to the development of mathematics.** ■ Recognize the contributions to the development of mathematics by women, men, and various cultures (e.g., what is the history of fractions?).

Component 5.2: Relate mathematical concepts procedures to other disciplines.

GLE	5	6	7	8	9/10
5.2.1	**Apply mathematical patterns and ideas in familiar situations in other disciplines.** ■ Find the mean from a given set of data using objects, pictures, or formulas. ■ Interpret skew, clusters, and gaps in given one-variable data displays. ■ Use estimation strategies and identify the reasonableness of answers. [1.1.8]	**Analyze mathematical patterns and ideas to extend mathematical thinking and modeling to other disciplines.** ■ Identify geometric figures and concepts in nature and art (e.g., triangle in architecture, rhombus in beadwork). [1.3.2] ■ Show the order of the set of integers on a number line with both positive and negative numbers (e.g., organize and graph on a number line the given birth years of the given Arabic kings). [1.3.3] ■ Read a micrometer to the nearest hundredth of an inch or centimeter, depending on the tool. [1.2.4] ■ Create a physical activity plan that results in 2500 calories expended over the week. ■ Calculate the ratio of various parts of an artwork (length of eyes to ears). ■ Discuss the difference between $^{3}/_{4}$ time and $^{6}/_{8}$ time and how it relates to a model.	**Analyze mathematical patterns and ideas to extend mathematical thinking and modeling to other disciplines.** ■ Evaluate and explain conclusions of plant growth drawn from data (e.g., from magazines, newspapers, web sites). [1.4.6] ■ Write a story about a situation that represents a given linear equation, expression, or graph. [1.5.2] ■ Determine the target heart zone for participation in aerobic activities. ■ Chart a one-week physical activity log based on calories expended/minute of activity. ■ Determine adjustments needed to achieve a healthy level of fitness. ■ Create a perspective drawing using vanishing point. ■ Mix paint in the correct proportions to create a particular color.	**Analyze mathematical patterns and ideas to extend mathematical thinking and modeling to other disciplines.** ■ Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken (e.g., age groups, regions of the U.S., genders, racial/ethnic distribution). [1.4.6] ■ Check to see if a corner is square using the Pythagorean Theorem. [1.2.5] ■ Calculate the one-repetition maximum for strength training of one muscle group. ■ Monitor/track a diet and evaluate the relationship to physical performance (e.g., does it meet daily nutritional requirements/energy for various populations and energy requirements based on lifestyle, safe-work practices, and leisure activities).	**Analyze mathematical patterns and ideas to extend mathematical thinking and modeling in other disciplines.** ■ Justify a prediction or an inference based on a set of data. [1.4.6] ■ Create a physical activity plan that results in a specified number of calories over a specified time. [PE]
5.2.2	**Know the contributions of individuals and cultures to the development of mathematics.** ■ Recognize the contributions to the development of mathematics by women, men, and various cultures (e.g., what is the history of probability theory?).	**Know the contributions of individuals and cultures to the development of mathematics.** ■ Recognize the contributions of a variety of people to the development of mathematics (e.g., research the concept of the golden ratio).	**Know the contributions of individuals and cultures to the development of mathematics.** ■ Recognize the contributions of a variety of people to the development of mathematics (e.g., research and report on the history of pi).	**Know the contributions of individuals and cultures to the development of mathematics.** ■ Recognize the contributions of a variety of people to the development of mathematics (e.g., research the history of the Pythagorean Theorem).	**Know the contributions of individuals and cultures to the development of mathematics.** ■ Recognize the mathematical contribution of a person or culture (e.g., create a report or presentation that highlights a mathematical contribution related to current mathematical study).

EALR 5: The student understands how mathematical ideas connect within mathematics, to other subject areas, and to real-life situations.

Component 5.3: Relate mathematical concepts and procedures to real-world situations.

GLE	K	1	2	3	4
5.3.1	**Understand how mathematics is used in everyday life.** ■ Generate examples of mathematics in everyday life: – counting (e.g., the number of people ahead of us in a line); – sorting things (e.g., grouping socks by color in order to match them up); – comparing things (e.g., who has the biggest piece of cake for dessert, or who is tallest/shortest in the family); – pointing out patterns (e.g., in clothing, fence posts, designs on buildings). ■ Identify objects based on a description of their geometric attributes (e.g., buildings have sides; some windows are shaped like a rectangle). ■ Describe the location of objects relative to each other (e.g., in, out, over, under, school bus stops next to each other).	**Understand how mathematics is used in everyday life.** ■ Generate examples of mathematics in everyday life: – counting (e.g., the pennies in the penny jar); – comparing measurements (e.g., standing up against the mark on the wall to check for growth); – building things (e.g., a snowman with three spheres, a dog house made of a box with a triangular roof); – playing games (e.g., when counting spaces on a board or knowing money is needed). ■ Describe familiar two-dimensional shapes based on their geometric characteristics (e.g., sharp corners, sides of different lengths). ■ Identify and sort two-dimensional shapes in their surroundings. ■ Skip count by 5s or 10s (e.g., with nickels or dimes).	**Understand how mathematics is used in everyday life.** ■ Generate examples of mathematics in everyday life: – counting (e.g., tallies to keep score during a game); – comparing lengths or distances where direct comparison is not possible (e.g., using a string or paper strip to compare the height and width of a desk to see if it fits in the room); – drawing geometric shapes (e.g., using a ruler to create shapes with equal sides). ■ Select the most appropriate unit to measure a given time (e.g., would you use minutes or hours to measure brushing your teeth, eating dinner, sleeping?); ■ Estimate the cost of two items knowing the approximate cost of one (e.g., one game costs about $8).	**Understand that mathematics is used in daily life and extensively outside the classroom.** ■ Write and solve multi-step situations that involve addition and subtraction. [1.1.6] ■ Use referents to standard units (e.g., width of pinkie finger is similar to a centimeter). [1.2.6] ■ Identify the point of final destination using real-world examples given directions for movement on a positive number line (e.g., travel back and forth on a street, temperature variation at different times of the day, climbing up and down stairs). [1.3.3] ■ Pose questions and gather data about self and surroundings. [1.4.2] ■ Create and solve a problem situation where mode is meaningful for a set of data. [1.4.4] ■ Make inferences on data from a real-world context, then use the context to determine if the inference is valid. [1.4.5]	**Understand that mathematics is used in daily life and extensively outside the classroom.** ■ Describe situations where area is the needed measurable attribute (e.g., the pricing of buying carpet, painting a wall, picking largest bedroom). [1.2.1] ■ Measure perimeter and area for regular and irregular shapes (e.g., use tiles, inches, or grid paper to find perimeter or area of blankets, CDs, skateboards). [1.2.2] ■ Identify situations in which estimated measurements are sufficient and use estimation to obtain reasonable measurements. [1.2.6] ■ Identify parallel and perpendicular lines in two-dimensional shapes and figures and in the environment. [1.3.1] ■ Identify the likelihood of events and use the vocabulary of probability (e.g., weather, simple games, if homework will be assigned). [1.4.1]
5.3.2					

Component 5.3: Relate mathematical concepts and procedures to real-world situations.

GLE	5	6	7	8	9/10
5.3.1	**Understand that mathematics is used in daily life and extensively outside the classroom.** ■ Identify angles in the environment (e.g., in architecture, furniture, nature). [1.2.1] ■ Identify types of angles in polygons on a plane and in the environment. [1.2.1] ■ Solve problems involving angle measurements in real-life situations (e.g., determine if a piece of tile will fit in a corner by measuring the angle). [1.2.3] ■ Determine whether a situation needs a precise measurement or an estimated measurement. [1.2.6] ■ Explain a series of transformations in art, architecture, or nature. [1.3.4]	**Understand that mathematics is used in daily life and extensively outside the classroom.** ■ Write and solve real-world problem situations to find sums or differences of decimals or fractions (e.g., explain how to find the change received from a $50.00 bill when a given amount of CD's and tapes with prices are bought). [1.1.6] ■ Calculate the ratio of bicycle gears.	**Understand that mathematics is used in daily life and extensively outside the classroom.** ■ Describe a situation where estimation is sufficient in real-life contexts. [1.1.8] ■ Use properties of polygons and circles to solve real-world problems (e.g., find the amount of fencing needed for a pasture). [1.3.2] ■ Compare the unit prices of various soft drinks.	**Understand that mathematics is used in daily life and extensively outside the classroom.** ■ Use estimation to predict or to verify the reasonableness of calculated results. [1.1.8] ■ Evaluate conclusions drawn from a set of data and support with evidence (e.g., from newspapers, web sites, opinion polls). [1.4.6] ■ Analyze data from a newspaper article to see if the conclusions are reasonable. ■ Research how coding and decoding has played a part in history.	**Understand situations in which mathematics can be used to solve problems with local, national, or international implications.** ■ Explain a method for determining whether a real world problem involves direct proportion or inverse proportion. [1.1.4] ■ Describe how changes in the dimensions of objects affect perimeter, area, and volume in real-world situations (e.g., how does the change in the diameter of an oil drum affect the area and volume). [1.2.1] ■ Represent situations on a coordinate grid or describe the location of points that satisfy given conditions (e.g., locate a gas station to be equidistant from given cities; locate a staking point to maximize the grazing area of a tethered goat). [1.3.3]
5.3.2		**Understand that mathematics is used within many occupations or careers.** ■ Explain or describe the mathematics necessary to get and perform in a particular job (e.g., complete a project that researches how mathematics is used in careers or occupations of interest). ■ Identify where in a particular career mathematics is used (e.g., police work — looking for patterns in fingerprints or crimes).	**Understand that mathematics is used within many occupations or careers.** ■ Explain how mathematics is used in careers or occupations of interest (e.g., complete a mathematically based project).	**Understand that mathematics is used within many occupations or careers.** ■ Explain how mathematics is used in careers or occupations of interest (e.g., complete a mathematically based project).	**Understand the mathematical knowledge and training requirements for occupational/career areas of interest.** ■ Select a career and research the mathematics necessary to get the job and the mathematics used in the job.

Glossary of Mathematics Terms

This is an abbreviated glossary. For the complete glossary, visit the website at: www.wednet.edu/mathglossary.

Absolute value: the numerical value of a number without regard to its sign; the distance of the number from 0 to the number line (e.g., the absolute value of 3 is 3, of –9 is 9, and of 0 is 0). The absolute value of –5 is written as $|-5| = 5$.

Acute angle: an angle which measures less than 90 degrees and greater than 0 degrees

Acute triangle: a triangle with three acute angles

Algorithm: a step-by-step method for computing (e.g., the addition algorithm that describes how to find the sum when regrouping, or the long division algorithm)

Angle: two rays that share an endpoint; named according to the number of degrees of its measure

Area: The *area* of a flat, or plane, figure is the number of unit squares that can be contained within it. The unit square is usually some standard unit, such as a square meter, a square foot, or a square inch.

Arithmetic sequence: a list of numbers in which the difference between any two adjacent numbers is the same. The first number in the list is called the initial value. The list 1, 3, 5, 7 … is an arithmetic sequence because the difference between any two adjacent numbers is 2. That difference is called the common difference.

Associative property of addition: The sum stays the same when the grouping of addends is changed (e.g., $(a + b) + c = a + (b + c)$ or $(30 + 4) + 20 = 30 + (4 + 20)$).

Associative property of multiplication: The product stays the same when the grouping of factors is changed (e.g., $(a \times b) \times c = a \times (b \times c)$ or $(2 \times 3) \times 4 = 2 \times (3 \times 4)$).

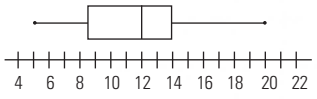
Attributes: a characteristic or distinctive feature

Average: a measure of central tendency; generally, *average* will imply arithmetic average, which could be the *mean*, *median*, or *mode*.

Axes: perpendicular lines used as reference lines in a coordinate system or graph; traditionally, the horizontal axis represents the *independent* variable and the vertical axis the *dependent* variable.

Bivariate data: data involving two variables, such as height and weight, or amount of smoking and a measure of health; often graphed in a scatter plot

Box-and-whisker plot: a graph which displays the following five points from a data set: the minimum value, the lower quartile (25th percentile), the median, the upper quartile (75th percentile), and the maximum value. The rectangle represents the middle 50% of the data, the vertical line in the rectangle represents the median, and the whiskers at both ends represent the remainder of the data. The endpoints on the whisker represent the smallest and largest values. In the example following, the median is 12, the 25th percentile is around 8.5, the 75th percentile is 14, the largest value is 20 and the smallest is 5.



Capacity: the volume of material or liquid that can be poured into a container

Cardinal number: a number that designates how many objects, or the number of units in the set; answers the question, "how many …?" (e.g., there are 28 students in the room; the cardinality or cardinal number is 28)

Central tendency: a single number that describes all the numbers in a set. The usual measures that are used are mean, median, or mode (e.g., for the set of numbers 95, 86, and 83, the mean is 89).

Cluster: in terms of statistics, a relatively large number of data that is closely grouped around a particular value

Coefficient: the numerical part of an algebraic term (e.g., 2 and 3 are coefficients in $2x + 3xy$)

Combination: a group of objects, numbers, or events; changing the order does not create a new combination (1, 2, 3 is the same combination as 3, 1, 2).

Composing numbers: the process of putting numbers together. When used here, it is implying that students look for numbers that could be combined easily to assist with calculation or understanding (e.g., 7 and 3, 24 and 26).

Composite number: an integer greater than 1 which has whole number factors other than itself and 1 (e.g., 10 is a composite number because it has the factors of 2 and 5, in addition to 1 and 10)

Compound event: an event that consists of two or more simple events (e.g., consider the event of rolling a six on a number cube and flipping a coin with a result of tails)

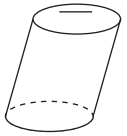
Conditional probability: the probability that an event will occur given that another event has already occurred

Congruent figures: figures that have the same shape and size

Conjecture: inference or judgment based on inconclusive or incomplete evidence; guesswork

Coordinates: an ordered pair of numbers that identify a point on a coordinate plane

Cylinder: a solid figure with two circular or elliptical bases that are congruent and parallel to each other



Decomposing numbers: the process of separating numbers into their components. This is normally done to enhance understanding or to simplify calculations (e.g., 123 can be thought of as 1 one-hundred, 2 tens, and 3 ones. 28 + 32 could be decomposed to 20 + 30 + 8 + 2).

Dependent event: an event whose probability is determined by the outcome of another event

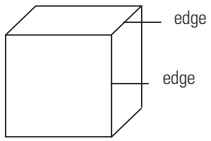
Derived unit of measure: a measurement determined by finding the ratio of other measurements (e.g., density is determined by dividing the mass of quantity by its volume; speed by dividing distance covered by time elapsed)

Direct proportion: Proportionality indicates that two quantities or variables are related in a linear manner. If one quantity doubles in size, so does the other; if one of the variables diminishes to $^1/_{10}$ of its former value, so does the other.

Divisible: One integer is divisible by another non-zero integer if the quotient is an integer with a remainder of zero (e.g., 12 is divisible by 3 because 12 ÷ 3 is an integer, namely 4).

Domain: set of all values of the independent variable of a given function, usually the x-values on a coordinate plane

Edge: the line segment formed by the intersection of two faces of a three-dimensional figure; a cube has 12 edges.



Empirical frequency: the number of times in an experiment that a particular event occurs

Empirical results: the results of an experiment or simulation

Equation: a number or algebraic sentence which shows equality between two sets of values (e.g., 4 + 8 = 6 + 6 or x + 4 = 8)

Event: any subset of the sample space. In rolling a number cube, the event of rolling a "3" is a singleton event because it contains only one outcome. The event of rolling an "even number" contains three outcomes.

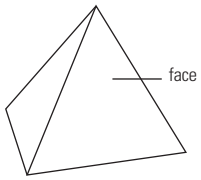
Expanded form: a number written in component parts showing the cumulative place values of each digit in the number (e.g., 546 = 500 + 40 + 6)

Experimental probability: the ratio of the number of times an event occurs to the number of trials

Expression: a variable combination of variables, numbers, and symbols that represent a mathematical relationship

Extrapolate: to estimate or approximate a value beyond a given set of data

Face: a flat surface, or side, of a solid figure



Flip: the effect of a flip is a reflection (see **reflection**)

Fluency: In number sense, fluency is defined by efficiency, accuracy, and flexibility.

Fundamental counting principal: If one event has m possible outcomes and a second independent event has n possible outcomes, then there are m x n total possible outcomes for the two events together.

Geometric sequence: a sequence of numbers, called terms, in which each successive term is determined by multiplying the previous term by a common factor (e.g., 1, 2, 4, 8, 16 … is a geometric sequence with a first term of 1 and a common factor of 2)

Histogram: a bar graph that shows the frequency distribution for a set of data; the graph is noted for the labels of the bars being given in intervals and for no spaces between successive bars.

Identity property of addition: Adding zero to a number gives a sum identical to the given number.

Identity property of multiplication: Multiplying a number by 1 gives a product identical to the given number.

Impossible event: an event that cannot happen or an event with a probability of 0

Independent events: two events whose outcomes have no effect on one another (e.g., the second flip of a coin is independent of the first flip of a coin)

Indirect measurement: a measurement determined without the direct application of measurement tools (e.g., finding a measure by the use of the Pythagorean Theorem, by similarity, or through ratios or scale factors)

Infer: to draw a conclusion from facts or evidence

Interpolate: to estimate or approximate a value between two given values

Inverse property of multiplication: Each non-zero real number x has a multiplicative inverse, denoted by $^1/_x$, such that their product is 1 (e.g., $^1/_3$ is the multiplicative inverse of 3).

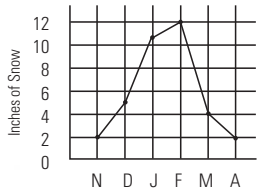
Irrational number: a number that cannot be written in a fraction form

Irregular polygon: a polygon whose interior angles are not equal and/or sides are not equal in length

Line of best fit: a line drawn on a scatter plot to estimate the relationship between two sets of data

Line graph: a graph that uses a line to show that something is increasing, decreasing, or staying the same over time

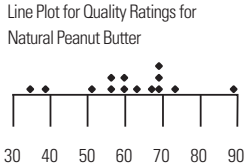
Amount of Snowfall by Month



Line of symmetry: a line on which a figure can be folded into two parts that are congruent mirror images of each other

Line plots: A line plot, sometimes called a dot plot, starts with a line that represents the variable you are interested in, and the values of the variable are labels on the line; each observation is marked as a point above the line.

Example of a line plot:



Linear or linear relationship: any data set or information that could be reasonably modeled with a line

Linear equation: an equation whose graph on a coordinate grid is a line and that can be written in the form $y = mx + b$

Linear inequality: an inequality whose graph on a coordinate grid is bounded by a line and that can be written in the form y ($\geq$, $<$, $>$, *or* $\leq$) $mx + b$

Mean: In the case of this document, *mean* implies arithmetic mean which is a measure of tendency found by summing all members in a set of data and dividing the number by members of the set. The arithmetic mean is often called the average (e.g., if there are three classes: $A = 24$ children, $B = 25$ children, and $C = 23$ children, the classes would be balanced by moving one student from class B to class C, thus making each class the same size (24). This number would indicate the average class size). Arithmetically, it is obtained by adding all data points together and dividing the sum by the number of points ($24 + 25 + 23 = 72$; $72 \div 3 = 24$).

Measures of central tendency (see **average**)**:** mean, median, mode

Median: the number in the middle of a set of data arranged in order from least to greatest or from greatest to least, or the average of the two middle terms if there is an even number of terms (e.g., *For the data:* 6, 14, 23, 46, 69, 72, 94 –> the median is 46 (the middle number) *For the data:* 6, 14, 23, 69, 72, 94 –> the median is also 46 (the average of the two middle numbers in the list))

Mode: the item that occurs most frequently in a set of data. There may be one, more than one, or no mode (e.g., the mode in {1, 3, 4, 5, 5, 7, 9} is 5).

Mutually exclusive: Two events are mutually exclusive if it is not possible for both of them to occur (e.g., if a die is rolled, the event "getting a 1" and the event "getting a 2" are mutually exclusive since it is not possible for the die to be both a one and a two on the same roll).

Non-linear: a data set or function that, when plotted, does not have the characteristics of a line

Non-standard units of measure: measurement units that are not commonly accepted as standard but are applied uniformly when measuring (e.g., paperclips, pencils, cubes)

Number line: a line that shows numbers ordered by magnitude from left to right or bottom to top; an arrowhead at each end indicates that the line continues endlessly in both directions; points are marked to subdivide the line into intervals that correspond to indicated numbers.

Number sentence: two or more expressions separated by a relational symbol (=, >, <, ≤, ≥); the relational symbol can be thought of as the verb in the sentence (e.g., 7 + 7 = 8 + 6; 14 < 92; 56 + 4 > 59).

Open-ended problem: a problem with different possible solution paths and which may have different solutions depending on the route taken

Order of operations: In simplifying an expression involving a number of operations, perform the operations in the following order:

1. complete all operations inside parentheses first;
2. calculate powers and roots and in the order they occur from left to right;
3. calculate all multiplications and divisions — left to right;
4. calculate all additions and subtractions — left to right (e.g., 7 + 3 x 8 = 31 [multiply 3 x 8 before adding 7]).

Ordered pair: two numbers for which their order is important when used to locate points on a coordinate graph; the first element indicates distance along the *x*-axis (horizontal) and the second indicates distance along the *y*-axis (vertical).

Ordinal number: a number that designates the position of an object in order; *first, second*, and *third* are examples of ordinal numbers (e.g., eraser is the SECOND element in the set (pencil, eraser, desk, chalkboard, book, file, paper); *Z* is the TWENTY-SIXTH element in the set (a, b, c, d … z)).

Outcome: one of the possible results in a probability situation or activity

Outlier: a number in a set of data that is much larger or smaller than most of the other numbers in the set

Parallel lines: lines that lie in the same plane and never intersect

Parallelogram: a quadrilateral with opposite sides parallel

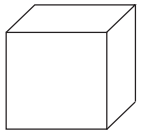
Perpendicular lines: lines that lie on the same plane that intersect to form right angles (90 degrees)

Pi (π): the Greek letter (π) that in mathematics represents the ratio of the circumference to the diameter of a circle; the value of pi is approximately 3.14159.

Pictograph: graph that uses pictures or symbols to represent similar data

Place value: the value of a digit as determined by its place in a number (e.g., in the number 135, the 3 means 3 • 10 or 30; in the number 356, the 3 means 3 • 100 or 300)

Polyhedron: a solid figure, the sides of which are polygons



Power: a term of the form x^n resulting from repeated multiplication of a factor (e.g., 16 or 2^4 is the fourth power of 2, since 2 has been used as a factor four times)

Precision: an indication of how finely a measurement is made; related to the unit of measurement and the calibration of the tool (e.g., was the measurement made using a ruler marked in increments of $\frac{1}{4}$ of an inch or in increments of $\frac{1}{16}$ of an inch)

Prime number: a whole number greater than 1 having exactly two whole number factors, itself and 1 (e.g., 7 is prime since its only whole number factors are 1 and 7)

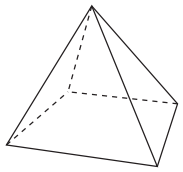
Prism: a three-dimensional figure that has two congruent and parallel faces that are polygons and the remaining faces are parallelograms

Probability: the numerical measure of the chance that a particular event will occur, depending on the possible events; the probability of an event is always between 0 and 1, with 0 meaning that there is no chance of occurrence and 1 meaning a certainty of occurrence.

Proportion: an equation showing that two ratios are equivalent (e.g., $\frac{2}{3} = \frac{6}{9}$)

Proportional: constituting a proportion; having the same, or a constant, ratio; as, proportional quantities

Pyramid: a solid whose base is a polygon and whose faces are triangles that meet at a common point (vertex)



Pythagorean Theorem: In any right triangle having a hypotenuse of length *c* and two legs of lengths *a* and *b*, $a^2 + b^2 = c^2$.

Random sample: a sample in which every person, object, or event in the population has the same chance of being selected for the sample

Range (functional): the set of all values of the dependent variable of a given function, usually the *y*-value on a coordinate plane

Range (statistical): the absolute value of the difference between the largest and smallest values in a set of data (e.g., the range of {2, 4, 6, 7, 9, 13} is 13 – 2 or 11)

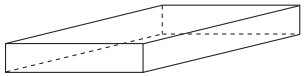
Rate: a ratio comparing two quantities measured in different units where one is measured in time (e.g., miles per hour and heartbeats per minute are rates)

Ratio: a comparison of two numbers using division (e.g., the ratio of two to five is 2:5)

Reasonable: within likely bounds; sensible (e.g., a reasonable estimate is close to the actual answer; an answer of 2 $\frac{1}{2}$ cans is not reasonable, while 2 cans or 3 cans is reasonable)

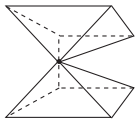
Reciprocal: the multiplicative inverse of a non-zero number (e.g., the reciprocal of *x* is given by $\frac{1}{x}$)

Rectangular prism: a polyhedron with six rectangular faces (e.g., the figure shown is a rectangular prism)



Reflection: a transformation of a figure by reflecting it over a line, creating a mirror image of the figure; the effect of a flip is a reflection.

Reflection on a point: a transformation of a figure by reflecting each of its points through a fixed point, called the center of the reflection, creating an image of the original figure across the center

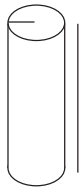


Regular polygon: a polygon with all sides having the same length and all angles having the same measure

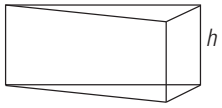
Right angle: an angle whose measure is 90 degrees

Right circular cylinder: a cylinder whose bases are circles and the centers of whose sections form a line perpendicular to the bases

Right cylinder: a cylinder with centers of whose sections form a line perpendicular to the bases



Right prism: a prism which has bases aligned one directly above the other



Right triangle: a triangle having one right angle

Rotation: a transformation of the points in a plane resulting from turning a figure about a specified point a fixed number of degrees or fractional portion of a turn either clockwise or counterclockwise

Sample space: a set of all possible outcomes to a specified event

Scale: sequenced collinear marks usually at regular intervals or else representing equal steps that are used as a reference in making measurements

Scale factor: a ratio that compares two sets of measurements such as the size of a model to the actual size of the object being modeled

Scatter plot: a graph of points (x, y), one for each item being measured, on a coordinate plane; the two coordinates of a point represent their observed, paired values (e.g., the ordered pairs may relate temperature to time of day).

Scientific notation: a number expressed in the form of $a \times 10^n$ where $1 \leq a < 10$ and n is an integer (e.g., 342.15 can be written in scientific notation as 3.4215×10^2)

Sequence: a set of numbers arranged in a special order or pattern

Similar figures: having the same shape but not necessarily the same size (congruent corresponding angles and proportional corresponding sides)

Similarity: characteristic of similar figures

Simulation (probability): using an experiment based on a real-life situation to answer a question (e.g., toss a coin to simulate true-false; heads = true, tails = false)

Single variable equation: an equation with one variable (e.g., $3x + 2 = 8$)

Single variable expression: an expression with one variable (e.g., $3x + 2$)

Single variable inequality: an inequality with a single variable (e.g., $3x + 2 > 8$)

Skip count: counting by groups as in skip count by 2s, 5s, or 10s; can be thought of as a precursor to multiplication

Slide translation: The effect of a slide is a translation.

Slope: the ratio of the change in vertical to the change horizontal between two points on a line (e.g., the slope of a line through (3,4) and (9,5) is $\frac{5-4}{9-3}$ or $\frac{1}{6}$)

Solid: a geometric figure with three dimensions

Square number: an integer that is a perfect square of another integer (e.g., 49 is the square of 7: that is, the product of a number multiplied by itself)

Square root: one of two equal factors of a given number (e.g., 7 is the square root of 49 because $7 \bullet 7 = 49$)

Standard form: a number written with one digit for each place value (e.g., the standard form for five hundred forty-six is 546; the standard form for three thousand six is 3,006)

Standard units of measure: units of measure commonly used, generally classified in the U.S. customary system or metric system (e.g., feet, meters, acres, gallons, liters)

Stem-and-leaf plot: a method of organizing data from least to greatest using the digits of the greatest place value to group data

Example: Ages of Adults in the Park

Data set				*Stem*	*Leaves*		
23	25	29	29	2	3	5	9 9
36	38	39	39	3	6	8	9 9
52	54	55	55	5	2	4	5 5

Successive events: events that follow one another in a compound probability setting

Surface area: the sum of the areas of all the faces of a three-dimensional object

Symmetrical: having a line, plane, or point of symmetry such that for each point on the figure, there is a corresponding point that is the reflection of that point (see **line of symmetry**)

System of equations: two or more equations in terms of the same variables; the solution of a system is a set of values for the unknowns (variables) that satisfies all the equations simultaneously.

Tessellate: to form or arrange polygons in a checkered or mosaic pattern

Theoretical probability: a measure of the likelihood that an event will occur; is equal to the ratio of favorable outcomes to the number of possible outcomes (e.g., knowing that there are six possible outcomes for rolling a fair number cube, one can assign the probability of $1/6$ to each of the possible outcomes)

Transformation (geometric): a change in position of a figure using a translation, reflection, rotation, or a combination of these mappings

Translation: a transformation of a figure by moving it without turning or flipping it in any direction; the effect of a slide is a translation.

Trapezoid: a quadrilateral that has two parallel sides

Trend: the general direction or tendency of a set of data

Univariate data: data that measures a single characteristic

Variability of data: Range, average deviation, standard deviation, and spread are all ways to describe the variability of data.

Variation (direct): a relationship between two variables that can be expressed in the form $y = kx$ where $k \neq 0$. $y = kx$ can be read as "y varies directly with respect to x."

Variation (inverse): a relationship between two variables that can be expressed in the $y = \frac{k}{x}$ where $k \neq 0$ $y = \frac{k}{x}$ can be read as "y varies inversely with respect to x."

Vertex: a point at which two lines meet to form an angle, where edges of a polygon or polyhedron intersect, or the point opposite the base in a pyramid or cone

Vertices: plural of vertex

Zero property of addition: Adding zero to a number gives a sum identical to the original number; zero is the identity element of addition (see **identity property**) (e.g., 4 + 0 = 4; 56.89 + 0 = 56.89).

Zero property of multiplication: The product of any number and zero is zero (e.g., 4 x 0 = 0; 0 x 456.7 = 0).

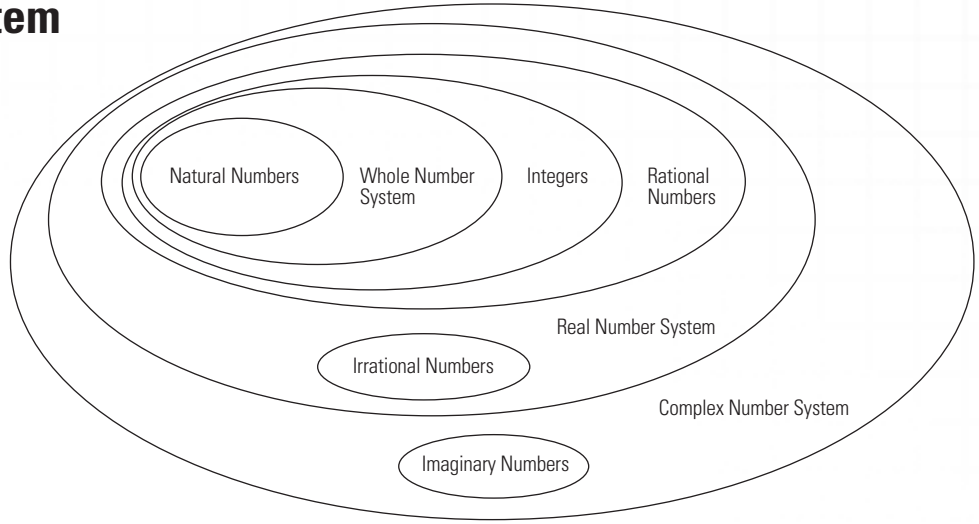
Cognitive Demand
Adapted from Bloom's Taxonomy of the Cognitive Domain

Cognitive Demand	Performance Terms
Knowledge: Recall — Remember previously learned materials. *Evidence of Learning* ■ Recognize the contributions of women, men, and people from a variety of cultures to the development of mathematics. (Grade 4: 5.2.2) ■ Recall addition and subtraction facts to at least 18. (Grade 2: 1.1.6) ■ State methods and procedures.	■ Match ■ Define ■ Repeat ■ Memorize ■ Label/Name ■ Outline/Format ■ Recount ■ Record ■ Recognize ■ Identify ■ Sort ■ List ■ State
Comprehension: Understand — Grasp the meaning of material: translate, interpret, extrapolate. *Evidence of Learning* ■ Identify the ordinal position of objects through at least tenth. (Grade K: 1.1.2) ■ Create a table or graph given a description of, or an equation for, a situation involving a linear or non-linear relationship. (Grade 7: 4.1.2) ■ Illustrate integer values using models and pictures. (Grade 6: 1.1.1)	■ Locate ■ Identify ■ Restate ■ Paraphrase ■ Describe ■ Summarize ■ Cite ■ Document/ Support ■ Infer ■ Predict ■ Illustrate ■ Show ■ Express ■ Explain
Application: Generalize — Use learned material in new situations. *Evidence of Learning* ■ Use counting strategies to combine numbers under 20. (Grade 1: 1.1.7) ■ Solve equations with an unknown. (Grade 2: 1.5.5) ■ Calculate measures of objects for which no direct information is given. (Grade 8: 1.2.5)	■ Select ■ Use ■ Manipulate ■ Organize ■ Imagine ■ Test ■ Demonstrate ■ Solve ■ Dramatize ■ Frame

Cognitive Demand	Performance Terms
Analysis: Breakdown — Break down material into its component parts so that it may be more easily understood. *Evidence of Learning* ■ Break down a situation in order to paraphrase it. (Grade 5: 3.1.1) ■ Determine if enough information is given to find a solution. (Grade 7: 2.2.1) ■ Determine whether the underlying model for a set of data is linear. (Grade 9–10: 1.4.5) ■ Justify the use of a chart or table to collect and organize information used to solve a problem (e.g., the two- or four-column chart helped to keep track of the information). (Grade 2: 3.2.3)	■ Examine ■ Classify ■ Research ■ Debate/Defend ■ Map ■ Characterize ■ Compare/ Contrast ■ Conclude/Draw Conclusions ■ Refute ■ Similarities/ Differences ■ Distinguish/ Differentiate ■ Relate to ■ Outline ■ Generalize
Synthesis: Compose — Put material together to form a new whole. *Evidence of Learning* ■ Propose possible factors that may influence a trend but not be reflected in the data. (Grade 9–10: 4.1.2) ■ Use the properties of two-dimensional and three-dimensional figures to solve mathematical problems (e.g., find the width of a river based on similar triangles; given a set of parallel lines, a transversal, and an angle, find the other angles). (Grade 9–10: 3.1.1) ■ Examine a set of data, research other sources to see if the data is consistent, find articles written to see if the data makes sense, to develop a logical point of view and to support that view. (Grade 9–10: 3.3.2)	■ Propose ■ Plan ■ Compose ■ Formulate ■ Design ■ Construct ■ Imagine/ Speculate ■ Create ■ Invent ■ Integrate
Evaluation: Judge — Judge according to a set of criteria stated by the evaluator. *Evidence of Learning* ■ Judge the appropriateness of inferences made from a set of data and support the judgment. (Grade 6: 1.4.6) ■ Judge conclusions drawn from a set of data and support with evidence. (Grade 8: 3.2.2) ■ Determine the accuracy and completeness of the data in a table or graph. (Grade 7: 1.4.6)	■ Evaluate ■ Judge ■ Weigh ■ Consider ■ Appraise ■ Recommend ■ Select the Best/Tell Why ■ Critique/ Criticize ■ Choose/Justify Choice ■ Scale

The Construction of Our Number System

The diagram to the right illustrates the organization of our number system. Each oval represents a set of numbers. One oval contained in another indicates that all of the numbers in the set are included in the numbers of the larger oval's set (e.g., the whole numbers include the natural numbers). Oval size does not represent the relative number of numbers in each of the sets.



The table below lists, describes, and gives examples of each of the sets of numbers included in the diagram.

Set	Description	Examples	Expected Introduction Grade
Natural numbers	Counting numbers.	{1, 2, 3, …}	PreK
Whole numbers	All counting numbers and 0.	{0, 1, 2, 3, …}	1st grade
Integers	All whole numbers and their opposites.	{…, –3, –2, –1, 0, 1, 2, 3, …}	6th grade
Rational numbers	All numbers that can be written as a ratio (fraction) of an integer over a natural number.	$\frac{3}{4}$, –300.73, 123, $-1\frac{7}{8}$, …	4th grade — positives 7th grade — negatives
Irrational numbers	All numbers that can't be written as a ratio of an integer over a natural number.	$\sqrt{2}$, $\sqrt[3]{17}$, π	9th grade
Real number system	All rational and irrational numbers.	Any numbers from the types above.	10th grade
Imaginary numbers	All numbers whose square is negative; or the square root of a negative number; any number that can be written in the form a + b*i*, where *a* and *b* are real numbers and *i* represents $\sqrt{-1}$.	$\sqrt{-6}$, $4\sqrt{-47}$, $3 + 2i$	11th grade
Complex numbers	All real and imaginary numbers.	Any number from the types above.	11th grade

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