Linear Algebra, MATH 2318

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Text: Elementary Linear Algebra with Applications, 3rd Edition, Richard Hill

Chapter 1	Introduction to Matrices
App 0.1	Twist, Tumble, and Travel to your Target, a Transformation game A linear algebra game introducing the notion of linear transformation. Image of a home is given, you are to find a set of movements that will transform it to a target image.
App 0.2	Why do we study mathematics? An application point of view A list of topics in a variety of fields that are built upon mathematics, including Linear Algebra.
Scroll 1	Section 1.1, Pages 1-3, Problems 1-20 Introduction, linear equations, a solution, solving, solution set, parametric solution, system of linear equations, linear systems, inconsistent systems, consistent systems, graphical solution, $0/1/\infty$ Theorem: a linear system may have none or one or infinitely many solutions.
App 1.1	Interactive Online Modules for Matrix Algebra Investigate the graphical solution of a 2×2 system in *The Geometer's SketchPad*.
Scroll 2	Section 1.1, Pages 4-9, Problems 21-32 Introduction, motivation for studying linear algebra, Gaussian elimination, triangular form, echelon form, back-substitution, coefficient matrix, augmented matrix, right hand side column, elementary operations on a linear system, elementary row operations on a matrix.
App 2.1	Linear Algebra Toolkit Row Operation Calculator allows you to perform or check many basic matrix operations.
App 2.2	Row Reducer Row Reduction Calculator performs the arithmetic of eliminations steps.
App 2.2	3-D Plotter Use this interactive app to see how the graph of a linear function, e.g., $z = 2x + 3y$, differs from a nonlinear one, e.g., $z = x^2 \cos(y)$.
Scroll 3	Section 1.2, Pages 10-13, Problems 1-14 Gaussian Elimination, parametric equations and deciding 0, 1, or infinitely many solutions, echelon form, leading zeros, pivot elements, leading/dependent variables, free/independent variables, how to solve an equation given in echelon matrix form.
Scroll 4	Section 1.2, Pages 13-21, Problems 15-21 Gaussian Elimination, Elementary row operations used to reduce an equation to its echelon form; so that it can be solved by back-substitution.

Chapter 1	Introduction to Matrices
Scroll 5	Section 1.3, Pages 23-27, Problems 1-14 Problem 15 from 1.2, matrix entry or element, matrix size or dimension, double subscript, matrix addition and subtraction, scalars and matrices, multiplication of a scalar and a matrix, dot product or inner product, matrix product as a collection of dot products.
App 5.1	Matrix Calculator Use this calculator to check your multiplications.
App 5.2	Interactive Online Modules for Matrix Algebra A visual demo of multiplication of 2×2 matrices.
App 5.3	Wolfram Demonstrations Project See how matrix product is constructed from the dot product of rows and columns.
Scroll 6	Section 1.3, Pages 27-29, Problems 15-16 Matrix product as a collection of dot products, general formula using summation notation.
Scroll 7	Section 1.3, Pages 30-31, Problems 21-32 Interpretations of matrix multiplication, multiplication by a diagonal matrix, a linear system written as matrix product, substitution as matrix multiplication, matrix multiplication as a sum of products of columns with rows.
Scroll 8	Section 1.3, Pages 32-34, Problems 33-42, Section 1.4, Pages 38-39 Different styles for matrix multiplication, interpretations of AB in terms of (a) dot products of rows of A with columns of B, (b) matrix A with columns of B, (c) rows of A with matrix B, (d) row matrices of A with column matrices of B, (e) sum of outer products of columns of A with corresponding rows of B, and (f) AX, with X a vector, as the linear combination of columns of A with weights same as entries of vector X.
Scroll 9	Section 1.4, Pages 38-45, Problems 1-14 Inverses and elementary matrices, the elementary matrix associated with multiplication of a row and exchange of two rows.
Scroll 10	Section 1.4, Pages 38-45, Problems 15-24, 36, 39, 40-43, 45, 49, 50 Inverses and elementary matrices Elementary *column* operations (related to problem 50). Problem 37 from 1.3. Different styles (row expansion, column expansion, outer product expansion) for multiplication of matrices, related to lecture 8. Elementary matrix associated with adding multiple of a row to another row. General procedure for finding the inverse.
Scroll 11	Section 1.4, Pages 48-49, Problems 9-20, 25-45 Finding Inverses using elementary matrices An example of 3×3 matrix being inverted. A short discussion of non-invertible matrices
Scroll 12	Section 1.5, Pages 57-62, Problems 1-8 LU factorization for a simple case (without permutation), description of algorithm and justification of procedure.
Scroll 13	Section 1.5, Pages 62-64, Problems 9-24 Solving $AX = B$ via LU factorization, advantages of LU factorization, solving systems with factorization, general permutation matrices, $PA = LU$ factorization.
Scroll 14	Section 1.5, Pages 64-67, Problems 33-36, Section 1.6 Pages 71-72, Problems 1-7, 13, 14,19, 20, Section 3.1 Pages 130-131 Solving $AX = B$ via $PA = LU$ factorization, an example, symmetric, skew-symmetric matrices, determinants of 2×2 and 3×3 matrices, vectors in physics

Chapter 3	Vectors
Scroll 15	Section 3.1, Pages 132-137, 151, Problems 1-30 with the exclusion of *span* questions Vectors, scalars, vectors, matrices, graphical addition, subtraction, and scalar multiplication of vectors
Scroll 16	Section 3.1, Pages 137-141, Problems 1-30 Norm(length, size, magnitude) of a vector, distance formula, dot product of vectors, angle between vectors, length in terms of dot product, dot product in terms of components, example: find the angle between two vectors.
Scroll 17	Section 3.1, Pages 140-144, Problems 31-42 Dot Product theorem, projections, Problems 5, 11, 36. Proof of Law of Cosines, Proof of $\mathbf{U} \cdot \mathbf{V} = u_1v_1 + u_2v_2 + u_3v_3 = \lvert\mathbf{U}\rvert\lvert\mathbf{V}\rvert \cos\theta$.
Scroll 18	Section 3.1, Pages 144-146, Problems 43-50 Gram-Schmidt Process, Problem 21, producing a set of mutually perpendicular vectors P_i out of an arbitrary vectors V_i using projections.
Scroll 19	Section 3.2, 3.3 Pages 149-165, Problems *all* Euclidean n-Space, General Vector Spaces, Closure.
Scroll 20	Review Sections, 1.4,1.5,1.6, 3.1,3.2 Inverse, LU factorization.
App 3.1	Interactive Online Modules for Matrix Algebra Learn about the linear combination, span, and linear independence by following the demo under Vector Spaces (third row, under construction).
Scroll 21	Section 3.4 Pages 165-169, Problems 1-24 Subspaces, closure with respect to vector addition and scalar multiplication, span.
Scroll 22	Section 3.4 Pages 165-169, Problems 1-24 Examples of subspaces.
Scroll 23	Section 3.4 Pages 169-173, Problems 33-40 Subspaces, null spaces, NS(A), linear combinations.
Scroll 24	Section 3.4, Pages 173-176, Section 3.5, Pages 179-180 Subspaces, linear combinations, in $AX = B$ vector B is a linear of columns of A with weight factor X , linear dependence and independence
Scroll 25	Section 3.5, Pages 179-180 Linear Independence, Problems 29 and 38 from 3.4.
Scroll 26	Section 3.5, Pages 180-185 Linear independence and dependence.
Scroll 27	Section 3.6, Pages 187-192, Problems 1-20 Basis, given a set of vectors how do we detect if they form a basis for a given space?

Chapter 5	Eigenvalues, Eigenvectors, and Eigenspaces
Scroll 28	Section 5.1, Pages 320-326, Problems 1-20 Determinants, brief review, short cuts, expansion/recursive formula
Scroll 29	Section 5.1, Pages 326-329, Problems 21-39 Determinants, three theorems
Scroll 30	Section 5.2, Pages 331-334, Problems 1-9, 11-22 (part b only) Introduction to eigenvalues and eigenvectors
App 30.1	Interactive Online Modules for Matrix Algebra Visual description of input and output vectors, eigenvalues and eigen vectors.
App 30.2	Interactive Mathematics Project Demo 1 gives a visual description of eigenvectors and eigenvalues.
App 30.3	Several videos for various cases of eigenvalues and eigenvectors.
App 30.4	Wolfram Demonstrations Project Visual and experimental way of finding eigenvectors and eigenvalues.
Scroll 31	Section 5.2, Pages 331-335, Problems 11-22 (parts a, b only) Eigenvalues (real, complex, double), characteristic polynomial of a matrix.
Scroll 32	Section 5.2, Pages 331-335, Problems 11-22 (parts a, b, c) Finding eigenvectors of a matrix, matrix factorization into *eigenvector matrix* $\times$ *eigenvalue matrix* $\times$ *inverse of eigenvector matrix*, $A = V\Lambda V^{-1}$
Scroll 33	Section 5.2, Pages 331-335, Problems 18-2 Finding eigenvectors of a 3×3 matrix
Scroll 34	Section 5.2, Pages 331-335, Problems 17-22 Diagonalization, $V^{-1}AV = \Lambda$, example of a 3×3 matrix
Scroll 35	Section 5.2, Pages 331-335, Problems 17-22 Repeated eigenvalues, finding eigenvectors and basis of eigenspaces for repeated roots of characteristic polynomial, several examples
Scroll 36	Section 5.2, Pages 338-339,, Problems 11-22 (part f), Section 5.3, Pages 342-349, Problems 1-26 Sum of eigenvalues=trace, product of eigenvalues=determinant, functions of a matrix, $f(A) = Vf(\lambda)V^{-1}$, diagonalizability.
Scroll 37	Section 5.3, Pages 342-349, Problems 1-26 Diagonalization, non-diagonalizable matrices
Scroll 38	Section 5.4, Pages 352-358, Problems 1-9 Symmetric Matrices, properties, orthogonal (orthonormal) matrices, Theorem: Eigenvalues of a real symmetric 2×2 matrix are real.
Scroll 39	Section 5.4, Pages 352-358, Problems 1-9 Review of symmetric matrices , Problem 1
Scroll 40	Sections 5.2, 5.3, 5.6 Eigenvalues, eigenvectors, diagonalization, a system of differential equations
Scroll 41	Sections 5.5, Pages 352-358 Markov processes