DISCRETE MATH: LECTURE 1

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1. In the Beginning...aka Chapter 1.1

1.1. The Trinity.

- A **Universal Statement** says that a certain property is true for all elements in a set. (*for all*)
- A **Conditional Statement** is an if-then statement, that is, if one thing is true then some other thing is also true. (*if-then*)
- A **Existential Statement** says that there is at least one thing for which the property is true. (*there exists*)

1.2. The Trinity Remix.

- **Universal Conditional Statements** are both universal and conditional.
  For example: For all animals a, if a is a dog, then a is a mammal.
  Your example:

- **Universal Existential Statements** are universal because the first part of the statement says that a certain property is true for all objects of a given type, and it is existential because its second part asserts the existence of something.
  For example: Every real number has an additive inverse.
  Your example:

- **Existential Universal Statements** assert that a certain object exists in the first part of the statement and says that the object satisfies a certain property for all things of a certain kind in the second part.
  For example: There is a positive integer that is less than or equal to every positive integer.
  Your example:

1.3. **In Class Group Work.** Section 1.1, Page 6: Answer questions 8, 10, and 12 and write down what kind of statement they are (i.e. is question 8 a universal statement, or a existential universal statement, or....).

2. Chapter 1.2 Sets

2.1. Organizing Math Stuff, or Sets if you want to be formal.

- A **Set** is a collection of objects. The objects in the collection are called the **Elements** of the set. If S is a set, then $\mathbf{x} \in \mathbf{S}$ means that x is an element of S. If we write $\mathbf{x} \notin \mathbf{S}$, we mean that x is not an element of S.
- Some sets in math are "celebrities", i.e. they are given special symbolic names that are used instead of set-roster notation or set-builder notation (more on set-builder below).
  - **R** or $\mathbb{R}$ stands for the *set of all real numbers.*
  - **Z** or $\mathbb{Z}$ stands for the *set of all integers.*
  - **Q** or $\mathbb{Q}$ stands for the *set of all rational numbers.*
- A set may be specified using the **Set-Roster Notation** by writing all of its elements between braces. For example: {1, 2, 3}

- NOTE: A set is completely determined by what its elements are–not the order in which they might be listed or the fact that some elements might be listed more than once.

Exercise: Let $A = \{1, 2, 3\}, B = \{3, 1, 2\}$, and $C = \{1, 1, 2, 3, 3, 3\}$. What are the elements of A? Of B? How about C? How are A, B, and C related?

Is $\{0\}$ the same as 0?

How many elements are in the set $\{1, \{1\}\}$?

- Another way to specify a set is called the **Set-Builder Notation**. Let S denote a set and let $P(x)$ be a property that elements of S may or may not satisfy. We may define a new set to be **the set of all elements** x **in** S **such that** $P(x)$ **is true**. We denote this set as: $\{x \in S | P(x)\}$
  For example: $E = \{n \in \mathbf{R} | n \text{ is a positive even integer}\} = \{2, 4, 6, 8, \ldots\}$
  Exercise: Write down the following set using Set-Builder Notation: X ={Jimmy Carter, George H.W. Bush, Bill Clinton, George W. Bush, Barack Obama}

2.2. Subsets.

- If A and B are sets, then A is called a **Subset** of B, written $\mathbf{A} \subseteq \mathbf{B}$, if, and only if, every element of A is also an element of B. It follows that for a set A to NOT to be a subset of set B means that there is at least one element of A that is not an element of B. Symbolically, $\mathbf{A} \nsubseteq \mathbf{B}$ means that there is at least one element such that $x \in A$ and $x \notin B$.
- Let A and B be sets. A is a **Proper Subset** of B, if, and only if, every element of A is in B but there is at least one element of B that is not in A.
- Set A and B are equal if, and only if, $A \subseteq B$ and $B \subseteq A$ are both true.
- NOTE: Do not confuse $\subseteq$ and $\in$! See Example 1.2.4 on page 10!
  For example: Let $A = \{1, 2, 3, 4, 5\}, B = \{1, 3, 5\}, C = \{2, 4, 5, 6\}$, and $D = \{5, 3, 1, 2, 4\}$.

2.3. Cartesian Products or Speed Dating for Mathematical Objects.

- Given elements a and b, the symbol (a, b) denotes the **Ordered Pair** consisting of a and b together with the specification that a is the first element of the pair and b is the second element. Two ordered pairs (a, b) and (c, d) are equal if, an only if, $a = c$ and $b = d$.
- Given sets A and B, the **Cartesian Product of** A **and** B, denoted $\mathbf{A} \times \mathbf{B}$ and read "A cross B," is the set of all ordered pairs (a, b), where a is in A and b is in B. Symbolically: $A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$.
  For example: Let $A = \{1, 2, 3\}$ and $B = \{u, v\}$. Then
  $A \times B$ is

$B \times B$ is

Let $\mathbf{R}$ denote the set of all real numbers. Describe $\mathbf{R} \times \mathbf{R}$.

2.4. **In Class Group Work.** Section 1.2, Page 13: Answer question 11.

3. Relations and Functions

3.1. **Relations: x and y hook up.**

- Let A and B be sets. A **Relation** R **from** A **to** B is a subset of $A \times B$. Given an ordered pair (x, y) in $A \times B$, x **is related to** y **by** R, written xRy, if, and only if, (x, y) is in R. The set A is called the domain of R and the set B is called its co-domain.
  For example: Let $A = \{2, 3, 4\}$ and $B = \{6, 8, 10\}$ and define a relation R from A to B as follows: For all $(x, y) \in A \times B$, $(x, y) \in R$ means that $\frac{y}{x}$ is an integer.
  Is $4R6$? Is $4R8$? Is $(3, 8) \in R$? Is $(2, 10) \in R$?

  Write R as a set of ordered pairs.

  Write the domain and co-domain of R.

  Draw an arrow diagram for R.

3.2. **Functions: x wants to be exclusive with y.**

- A **Function** F **from a set** A **to a set** B is a relation with domain A and co-domain B that satisfies the following two properties:
  (1) For every element x in A, there is an element y in B such that $(x, y) \in F$.
  (2) For all elements x in A and y and z in B, if $(x, y) \in F$ and $(x, z) \in F$, then $y = z$.

  For example: Let $A = \{2, 4, 6\}$ and $B = \{1, 3, 5\}$. Which of the relations R, S, and T defined below are functions from A to B?
  $R = \{(2, 5), (4, 1), (4, 3), (6, 5)\}$

  For all $(x, y) \in A \times B$, $(x, y) \in S$ means that $y = x + 1$.

  T is defined by the arrow diagram

3.3. **In Class Group Work.** Section 1.3, Page 21: Answer questions 3 and 7.